Output and Inflation in an Active-Fiscal, Passive-Monetary HANK

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Abstract

When fiscal policy is active and monetary policy is passive in a heterogeneous agent New Keynesian (HANK) model, deficit-financed transfers to low-asset households lead to similar cumulative inflation but greater increases in real output than transfers to wealthier households. Household heterogeneity and targeted policy change the timing of output gaps, making this consistent with the Phillips Curve, contrary to conventional "sacrifice ratio" intuition. Equilibria where fiscal policy is active or passive but slow adjusting are quantitatively similar, so long as monetary policy is passive. However, even large active fiscal transfers to high marginal propensity to consume households induce a less sustained output expansion than a monetary policy shock in a conventional active monetary/passive fiscal setting, as the former's effect on real output is more persistent. In contrast, when monetary policy is passive, monetary policy is roughly as stimulative as fiscal policy.

Keywords: fiscal theory, heterogeneity, inflation, HANK

JEL: E63, E31, E12

1. Introduction

The trade-off between real output and inflation following unanticipated changes to fiscal policy remains a long-standing open question in macroeconomics. In the parlance of Leeper (1991), much of the previous literature has focused on models where monetary policy is "active" and fiscal policy is "passive." However, monetary policy in the United States has been constrained by the zero lower bound for much of the early 21st century, while fiscal authorities have increasingly responded to changing macroeconomic conditions with tax cuts and transfer programs. As such, this paper departs from standard policy regime assumptions and instead explores the implications of active fiscal and passive monetary policy for output and inflation in a canonical heterogeneous agent New Keynesian (HANK) model with idiosyncratic income risk and incomplete asset markets, such that

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households are heterogeneous in their marginal propensities to consume (MPCs).

Following a fiscal expansion, the behavior of households with assets is key to determining the path of inflation, while the path of output is more strongly influenced by those who do *not* have assets and instead have high MPCs. If the government sends deficit-financed transfer payments to low-wealth households with high MPCs, then the cumulative increase in real GDP is predictably larger than when the transfers are sent to wealthier, lower-MPC households. However, the long-term effect on the price level, total cumulative inflation, is largely the same under both transfer policies provided they lead to similar amounts of nominal government debt (net nominal private assets) that are not paid off by future tax revenue and are instead inflated away.

This finding is contrary to popular intuition that output gaps and the price level move proportionally as measured by constant "sacrifice ratios," the percentage change in real GDP associated with a one percentage point abatement in inflation. Ball (1994) provides a classic survey describing empirical estimates of these ratios, which are often assumed to be tightly related to the inverse of the slope of the Phillips Curve. However, I show that for the New Keynesian Phillips Curve, the timing of the output gaps (an endogenous object) is crucial for determining the size of sacrifice ratios as well. Fast expansions or contractions move inflation less overall than slower ones, even with the simplest New Keynesian Phillips Curve; how a policy interacts with MPC heterogeneity changes this timing, and thus changes the trade-off. The trade-off in an active fiscal/passive monetary setting is furthermore very different from the behavior of a HANK under a standard passive fiscal/active monetary regime, where the central bank's inflation target is the key determinant of the path of inflation.

Different transfer programs can produce similar amounts of cumulative inflation and very different real GDP responses and be consistent with the Phillips Curve – but why would an active fiscal model generate such an equilibrium? For incomplete market models, the answer is related to the ideas conveyed in Hagedorn (2016, 2023, 2024) and the fact that policy and households are non-Ricardian. If the government issues more nominal debt, then the price level adjusts to equate the real demand for those assets to their real supply, via what Hagedorn (2016) calls the "demand theory of the price level" (DTPL). If prices take time to adjust and the real stock of private wealth increases, or if the distribution of risk-sharing otherwise better insures households, then households' precautionary savings motives fall and aggregate demand and consumption expenditures increase. Inflation then slowly erodes the nominal liquid balances held by households until real private assets in the economy return to steady state levels. If inflation overshoots, as it may in high-inflation episodes when transfers are distributed to high-MPC agents, then the paucity of real balances strengthens households' precautionary savings motive, lowering aggregate demand for marginal savers and inducing deflation until the overshoot is reversed. Since inflation stabilizes liquid assets and government debt, inflation predictably accumulates to inflate away government deficits not paid off by future taxes; the mechanism is technically distinct from the fiscal theory of the price level (FTPL) discussed in Cochrane (2023), but the two theories yield similar qualitative results, as I show via simulations in a two-agent setting.

However, while active fiscal policy can be targeted to send transfers to high-MPC households, monetary policy in an active monetary/passive fiscal framework is many times more stimulative – at least, in the world of rational expectations models. In an active fiscal scenario, transfers amounting to 1% of GDP directed to the lower half of the income distribution yield output gaps that rapidly accumulate to roughly 0.73% of annual steady state real GDP when integrated over time. If monetary policy is passive and acts through the forward guidance wealth effect channel described in Cochrane (2018), a 1% reduction in nominal rates with a half-life of four quarters generates an output response similar to that of targeted fiscal policy. However, in an active monetary setting the same interest rate shock generates a total sum of the output gaps that is nearly four times as large as the one generated by active fiscal policy. This is due to the gaps' persistence, which results from the long-lasting low interest rates and automatic transfers that a passive fiscal/active monetary policy mix induces.

Well-known papers like Kaplan et al. (2018) have noted that the fiscal adjustments following a monetary policy shock can have large implications for the impact of monetary policy. However, I show that the large effect does not simply come about because of the automatic fiscal adjustments passive fiscal policy entails; a similar equilibrium where *both* monetary and fiscal policy are passive does not generate this effect. Rather, the phenomenon is specifically a feature of the equilibrium selected by an active monetary policy Taylor rule. Active monetary policy depresses real interest rates for longer than passive monetary policy, facilitating greater fiscal expansion.

My analysis comes in three parts. First, I briefly outline the two leading theories regarding determinacy in models with active fiscal policy: the FTPL and the DTPL. I describe their similarities and differences and clarify that the DTPL determines the price level in my HANK model, in agreement with Hagedorn (2024). Second, I briefly outline a simplified two-agent New Keynesian (TANK) model where one group of households smooths consumption with their savings while the other group is constrained to spending their income as soon as it is received. I then provide closedform analytical results for the simple economy to ascertain why heterogeneity is of only minor relevance for the determination of the overall price level but important for the output response, and how this is consistent with the Phillips Curve.

In the third part of my analysis, I replace the two-agent block of the TANK model with a calibrated distribution of households over asset and income states. Agents face uninsurable idiosyncratic income risk and incomplete markets, yielding a canonical HANK framework with active fiscal policy and passive monetary policy. Unlike in the TANK model, the distributions of assets and MPCs are endogenous and targeted to match empirical moments, while the income distribution is parameterized to match data measurements of earnings autocorrelation and volatility. Although more complicated than the TANK setting, this model delivers similar conclusions. However, the added realism of asset and income inequality, precautionary savings motives, and endogenous MPCs make the setting an ideal "laboratory" with which to examine how active fiscal policy regimes function when fiscal transfers are targeted to one group but not another.

1.1. Related Literature

This paper is the first to numerically solve a calibrated incomplete markets business cycle model with New Keynesian nominal rigidities to evaluate the effects of fiscal policy when fiscal policy is active and monetary policy is passive. Although Hagedorn (2023, 2024) investigate the determinacy properties of such settings and contrast them with the fiscal theory of the price level (FTPL) – a topic which I discuss further in my next section – my paper extends this work to examine new perturbations from the non-stochastic steady state caused by targeted transfer payments. In particular, my work examines how targeted transfers to subgroups in the population have different implications for the trade-off between output and inflation. It is also the first to examine how the magnitude of the stimulus induced by a monetary policy shock in a HANK setting depends less on whether fiscal policy is active or passive and more on whether monetary policy is active or not, using a scenario where both types of policy are passive as a point of comparison. While my combination of elements is novel and relevant to our understanding of economic policy in the real world, the building blocks I use are drawn from the existing literature.

As alluded to in the introduction, I use the terms "active" and "passive" to describe fiscal and monetary policy in the style of Leeper (1991). "Active" fiscal policy pertains to fiscal policy that does not automatically stabilize a government's real debt to steady state levels over time for all sequences of the price level. Rather, changes in the price level stabilize real government debt in equilibrium, either immediately or through changes in inflation and the real interest rate. This is possible provided that debt is nominal and the central bank does not raise real interest rates in response to inflation – such that monetary policy is "passive." A passive fiscal/active monetary policy regime entails the converse: the government balances its budget to stabilize debt for every possible price level, while the central bank commits to raising real rates in response to inflation.

Most previous incomplete-market HANK models, like those pioneered by McKay et al. (2016), Kaplan et al. (2018), and Auclert et al. (2018, 2023b), and many others, use a passive fiscal/active monetary policy mix, unlike the active fiscal/passive monetary one that I explore. Even so, these preceding papers also characterize the high and low MPCs that result from ex-post household heterogeneity as key determinants of the response of employment and output to shocks at business cycle frequencies. My heterogeneous agent model's non-stochastic steady state is particularly reminiscent of McKay et al. (2016) and has a similarly calibrated idiosyncratic income process for the household block. However, Werning (2015) and Acharya and Dogra (2020) note that the cyclicality of income risk is a crucial factor for model dynamics and determinacy, making models highly sensitive to the distribution of corporate profits and taxation over the business cycle. To abstract away from these factors, which can be contested and difficult to measure, my baseline specification does not feature cyclical variation in corporate markups or real wages, nor does it feature a government that makes large automatic tax adjustments to balance the budget. Because my setting is close to the acyclic income risk environment of Werning (2015), the effects of monetary policy shocks in incomplete markets are very similar to those encountered in complete markets. As such, I keep my focus on fiscal policy shocks in this article, unlike the majority of the aforementioned HANK papers, but I ensure the monetary policy shocks that I compare fiscal expansions to are also not being amplified or dampened by income risk cyclicality.

Active fiscal policy with passive monetary policy has also been studied extensively in previous work, but largely in the context of the fiscal theory of the price level (FTPL) with representative agent models and complete market economies; this body of knowledge is surveyed extensively in Cochrane (2023). As such, most of these previous models do not discuss the way active fiscal policy can engage with economies that feature inequality, idiosyncratic income risk, borrowing constraints, and resulting MPC heterogeneity. However, I do begin with TANK models to describe the forces at work in the HANK model, and so there is some overlap between my paper and Bianchi et al. (2023); they fit a TANK model exhibiting the FTPL as well and find that MPC heterogeneity does little to change the path of prices following a fiscal stimulus relative to a representative agent New

Keynesian (RANK) model. Even so, their paper does not examine the total GDP response relative to the response of inflation, nor do they look at targeted stimulus payments made to subgroups of the population as mine does. They additionally set the fraction of hand-to-mouth households to just 7% of the population, leading their TANK model to have very small MPCs relative to the HANK literature's benchmarks (see Auclert et al. (2018)). They also focus primarily on the FTPL; I consider other active-fiscal price determination mechanisms as well.

While my analysis is the first to study active fiscal policy wherein one group receives transfers and others do not in a fully-fledged HANK model with nominal rigidities, Kaplan et al. (2023) has also made the important step of combining an active fiscal/passive monetary policy mix with a setting that includes incomplete markets and household heterogeneity, but no nominal rigidities. In a series of numerical experiments in an endowment economy, they show that a one-time fiscal helicopter drop in a heterogeneous agent setting produces more inflation in the short-run than in the representative agent model, as the transfers change the distribution of risk in the economy by moving resources to households at or near their borrowing constraints. The effect is transitory, however, and over time the price level in the heterogeneous agent model converges to the one-time price level jump experienced in the representative agent one. Most pertinently, whether the transfer is directly targeted to the poor or not plays only a relatively small role in their model's inflation dynamics – a property that I show is preserved in a setting with endogenous demand-determined output and sticky prices.[1] Given the difference in focus, but with the shared interest in describing active fiscal policy in heterogeneous agent settings, my analysis should be read as complementary to theirs.

All of my simulations are for certainty-equivalent models using linearized perturbations from a non-stochastic steady state. I use the sequence-space Jacobian technique of Auclert et al. (2021) to solve the HANK model. I also use the state-space method of Bayer and Luetticke (2020) (a modification of Reiter (2009)) solved via a Schur decomposition as an added numerical determinacy check. All models are solved using finite difference approximations in continuous time.

[1] Kaplan et al. (2023) additionally finds that while a heterogeneous agent fiscal theory economy enjoys uniqueness and determinacy when the government runs surpluses in the steady state, multiple equilibria may emerge when the government runs perpetual deficits and $r < g$. The authors suggest policy rules for eliminating this multiplicity of equilibria and run most of their simulations in an $r < g$ setting, but I consign my model to a more theoretically conventional environment with positive steady state primary surpluses and $r > g$.

2. Determinacy and the Fiscal and Demand Theories of the Price Level

The fiscal theory of the price level, as described by Cochrane (2023), states that when fiscal policy is active, the price level can be determined to equate the real market value of government debt with the future surpluses to which they are a claim. If q_t is the nominal price of nominal bonds $\tilde{B}_t$, p_t is the price level, and real surpluses are taxes less expenditures $T_t - G_t$, the central FTPL equation takes the form of equation (37) from Cochrane (2018):

$$\frac{q_t \tilde{B}_t}{p_t} = \mathbb{E}_t \int_t^{\infty} e^{-\int_0^{\tau} r_s ds} (T_\tau - G_\tau) d\tau \qquad \text{(Cochrane (2018), 37)}$$

This equation – derived by integrating forward the government's budget constraint and applying a transversality condition – holds in equilibrium for any model where there is no bubble value to government debt and real debt explosions are infeasible. A passive fiscal rule pays off government debt with new taxes by choosing sequences of $(T_t, G_t)_{t \geq 0}$ such that the above equation holds for *any* price level p_t, including off-equilibrium ones; the equilibrium price level is not determined by the above equation and the FTPL does not apply. However, if the government does not commit to responding to the price level by adjusting future surpluses, then fiscal policy is active. If the path of $(r_s)_{s \geq t}$ also does not adjust to exactly offset changes in p_t, such that the right-hand side of the equation does not automatically respond to exactly offset changes p_t, then the price level is determined since $\tilde{B}_t$ is set by the government and q_t is determined by the expected path of nominal interest rates.

However, complications arise when real interest rates are endogenous in the steady state of the economy, as is the case in OLG models. Farmer and Zabczyk (2019) and Hagedorn (2024) note that a continuum of initial price levels can satisfy the fiscal theory equation even when fiscal policy is active when real rates of return can adjust with the price level. In canonical incomplete markets models, so too are real interest rates endogenous. Kaplan et al. (2023) describe their model as the FTPL combined with heterogeneous agents, but Hagedorn (2024) argues against this interpretation and instead claims that the price level in incomplete market environments is determined by a "demand theory of the price level" (DTPL), as outlined in Hagedorn (2016). In this alternative interpretation, the real interest rate and real bond holdings are determined so as to equate agents' real asset demand (a downward-sloping function of the real interest rate) with the net supply of government debt to asset markets. If total net private nominal assets are set by policy, then this determines the price level – such that the prevailing price level is the one that clears the real asset market, a different equilibrium determination mechanism. Equivalently, the price level is the one that equates the real supply of goods with the real value of nominal demand. As evidence that the DTPL determines the price level and not the FTPL, Hagedorn (2024) notes that the price level in incomplete markets is still determinate even when *both* fiscal and monetary policy are passive.

My analysis concurs with the DTPL view. I check the determinacy of my model with three different measures: Blanchard and Kahn (1980) state-space model eigenvalue counting, along with Onatski (2006) winding number criteria like Auclert et al. (2023a) and Hagedorn (2023). These tests agree that my model is still determinate even when *both* fiscal and monetary policy are passive – such that the DTPL is determinant of the price level, not the FTPL. However, there are strong similarities between the DTPL and FTPL mechanisms. In a DTPL world, if the price level did not adjust to eventually inflate away new nominal balances following a deficit-financed fiscal stimulus, then the persistent wealth effect of the real assets would lead real consumption and debt to explosively diverge rather than return to steady state.

Conversely, in the FTPL world, if new government nominal debt is not inflated away, then government debt again rolls over into unsustainable levels, violating households' transversality constraint. In both theories, as Hagedorn (2024) notes, the growth of nominal debt is a "sufficient statistic" for the long-term growth in the price level. Using experiments with a TANK model in Appendix C, I show that a setting where DTPL determines the price level does not look that different from a setting where FTPL determines the price level. It is also the case in both the HANK and TANK models, the DTPL equilibria do not look markedly different when fiscal policy is active or just barely passive, so long as monetary policy is also passive; inflation's role in stabilizing the real value of nominal assets and liabilities is the key force in explaining how much the price level eventually moves in response to a fiscal shock.

3. Analytic Expressions from 4 Equations in a TANK Model

To fix ideas, I sketch a simple TANK model to intuit how transfers to high MPC agents might yield more output but very similar levels of inflation compared to transfers to low MPC households when fiscal policy is active and monetary policy is passive. The simple model includes two households, a government that can send exogenous transfers to each by running deficits and borrowing, and a basic New Keynesian Phillips Curve. For simplicity, I here consider a model that establishes determinacy using the FTPL, while my HANK model's determinacy comes from the DTPL. However, I parameterize and simulate a slightly more complicated TANK model in Appendix C that can display either a DTPL equilibrium or an FTPL equilibrium and show that the results for both are qualitatively similar.

The first representative household is a measure $1-\mu$ continuum of forward-looking agents who collectively hold the stock government debt (a "saver" household, labeled 1), and is of measure $1-\mu$. These households receive transfers $M_{1,t}$ from the government, which are equal to zero in the non-stochastic steady state, and additionally pay the government's steady state interest expense, T_{NSS}. They choose consumption $c_{1,t}$ in accordance with an Euler equation, derived in Appendix A, where ρ is the rate of time discounting, γ^{-1} is the elasticity of intertemporal substitution, and r_t is the real interest rate:

$$\frac{\mathbb{E}_t[dc_{1,t}]}{dt}\frac{1}{c_{1,t}} = \gamma^{-1}\left[r_t - \rho\right]. \tag{1}$$

The second, of mass μ, is a continuum of households who are constrained to consume their income every period. These agents ("spenders," labeled 2), set their consumption $c_{2,t}$ equal to labor income from working plus income from transfers they receive from the government.

$$c_{2,t} = Y_t + M_{2,t} \tag{2}$$

where $Y_t = L_t$ is aggregate output and hours worked, and the real wage rate is equal to 1 – as would be the case in a model where wages are nominally rigid and the output price sector is perfectly competitive, making the real wage perfectly acyclic.

The government issues nominal bonds of real value B_t at a real interest rate of r_t to pay for deficits. Tax revenue is $T_t = T_{NSS} - \frac{1}{1-\mu}M_{1,t} - \frac{1}{\mu}M_{2,t}$, where $T_{NSS} = r_{NSS}B_{NSS}$. As such, the real stock of government debt evolves according to

$$\frac{dB}{dt} = -T_t + r_t B_t \tag{3}$$

and the central bank fixes the nominal interest rate, such that $i_t = i$. With the Fisher equation, this means that $r_t = i - \pi_t$, where π_t is the rate of inflation.

The New Keynesian Phillips Curve is then

$$\rho\pi_t = \frac{\mathbb{E}_t[d\pi]}{dt} + \nu\widehat{Y}_t \tag{4}$$

where $\widehat{Y}_t$ is the percent deviation of real GDP from its value in the steady state (the output gap) ν is the slope of the Phillips Curve.

Suppose now that the economy is in steady state (with zero inflation and no transfers besides the lump-sum ones used to balance the budget) at time t when the government announces that it intends to send transfers to one household but not the other by temporarily raising either $M_{1,t}$ or $M_{2,t}$ from their steady state values by running deficits that will never be repaid with future taxes. After a short period of time, these deficits return to zero. To analyze such an experiment, I consider the equations one-by-one.

3.1. The Government Debt Equation

The government debt equation is particularly important to analyze here, as it is nearly identical to the aggregate equation that appears in my full HANK model later. If the price level does not jump on impact at time t and if nominal interest rates are held constant, then I can log-linearize the government debt equation to show

$$\mathbb{E}_t \int_t^{\infty} e^{-(\tau-t)r} \widehat{\pi}_\tau d\tau = -\mathbb{E}_t \left[\frac{T}{B} \int_t^{\infty} e^{-(\tau-t)r} \widehat{T}_\tau d\tau \right] \tag{5}$$

where variables without a time subscript denote values in the non-stochastic steady state, while hatted variables denote percent changes thereof. The steps to the derivation are provided in Appendix A.2.

Up to a first-order approximation, the present value of inflation (discounted according to the steady state discount rate) will be equal to the discounted value of future unfundend deficits as a percentage of steady state debt, as inflation is the force that stabilizes the debt. If the deficits are exogenous, then the only way that household heterogeneity enters into the above calculation is by affecting the timing of inflation, which changes how it is discounted. These timing effects are very small, however, if steady state interest rates are small ($r = 0.005$ in a quarterly calibration that targets 2% annual rates) and inflation mean reverts quickly after a few years. Even so, if inflation peaks rapidly following the transfer shock, as it might during a rapid output expansion following surprise transfers to high-MPC households, then slightly *less* cumulative inflation will follow than if the inflation had taken more time to arrive; the price level does not have to inflate away as much interest expense on top of the primary deficits to bring real the real value of nominal balances back to steady state levels.

Note that this logic depends only on the budget equation and the requirement that real bonds do not follow an explosive path and return to steady state. It holds for representative agent, two agent, and heterogeneous agent models with sticky prices and infinite planning horizons, regardless of the heterogeneity in the economy or the source of the nominal rigidities.

3.2. Households and Aggregate Demand

Aggregate demand in the economy is the sum of saver consumption plus spender consumption:

$$Y_t = (1-\mu)c_{1,t} + \mu c_{2,t}$$

Spender households have an MPC of exactly one when they receive transfers. In the appendix, I show that the saver households have a contemporaneous MPC of ρ out of a unitary increase in the present value of their expected lifetime income or their liquid assets; this is essentially the continuous time equivalent of the behavior described in Auclert et al. (2018), yielding an economy-wide average contemporaneous MPC out of a uniform transfer of $\mu + (1-\mu)\rho$. In a standard quarterly calibration, ρ is again on the order of 0.005. As such, in partial equilibrium (considering only the policy functions of the households and holding household labor income constant), transfers to spenders boost aggregate demand much more than transfers to savers, ceteris paribus.

3.3. How is this consistent with the Phillips Curve?

The Phillips Curve describes the dynamic relationship between inflation and the output. However, even in this very simplified model, the relationship between total *cumulative* inflation and output depends on the timing of the output gaps. In Appendix A.1, I show that integrating (4) forward twice yields

$$\int_t^{\infty} \pi_\tau d\tau = \nu \int_t^{\infty} (\tau - t) e^{-\rho(\tau-t)} \widehat{Y}_\tau d\tau \tag{6}$$

If the rate of discounting or amount of time since the shock has transpired is small, output gaps twice as far into the future count roughly double toward the total amount of inflation; the further the output gap is into the future, the more inflationary it is. To provide intuition, if a policy shock causes output gaps to jump and then decay back to zero at a constant rate of λ_Y, the ratio of cumulative inflation to cumulative output is $\nu\lambda_Y/(\lambda_Y+\rho)^2 \approx \nu/\lambda_Y$. Conversely, the implied cumulative sacrifice ratio is inversely proportional to the slope of the Phillips curve, and also directly proportional to the speed at which the output gaps are realized and decay.

The intuition is straightforward. It is true that inflation at time t jumps higher when current and future output gaps jump higher, all else equal. However, if firms or workers and unions take time to adjust their prices, then they are limited in how much they can immediately raise their prices in response to an acute surge in output. Additionally, they are forward-looking, so past output and inflation are sunk; only future output gaps matter for how they set prices. If real GDP returns to its steady state value quickly, these future output gaps may be small, even if past output gaps have been large. In this sense, price-setters in the economy tend to fall "behind the curve" for the transitory-but-potent real GDP expansions that transfers to high hand-to-mouth agents generate. By the time the economy returns to steady state, cumulative real output can rise higher for the same rise in the price level when it rises faster.

4. A Heterogeneous Agent New Keynesian (HANK) Model

In the full HANK model, time $t \geq 0$ is continuous. At a high level, the economy is populated by households who have the same preferences, but face borrowing constraints and different paths of idiosyncratic labor income shocks that they cannot fully insure. These households save by holding long-lived nominal government bonds and supply their labor to the market via decentralized unions. The output sector is perfectly competitive; wages adjust with nominal rigidities, such that labor demand and output are demand-determined. The government issues debt to pay for transfer payments and does not necessarily raise taxes to keep the debt from growing exponentially. A central bank sets nominal interest rates according to either a Taylor rule or an interest rate peg. The numerical solutions are all for a perfect foresight environment linearized around a nonstochastic steady state (NSS); once the shock is realized, the transition dynamics are deterministic and known to the agents in the model.

4.1. Households

A measure 1 continuum of households inhabit a Bewley-Aiyagari setting where they have two dimensions of ex-post heterogeneity: their labor-augmenting productivity z (generating income inequality) and their real asset position a (which agents endogenously determine based on their consumption choices). Households choose their consumption choice c with an intertemporal elasticity of substitution of $1/\gamma$ and supply hours worked h according to a rule set by unions, in so doing incurring labor disutility with a Frisch elasticity of η. The government taxes labor income at a fixed rate of τ. Bonds are nominal and trade at a nominal price of q_t. For convenience, I write a as assets valued at steady state bond prices q_{NSS}, such that $\frac{q_t}{q_{NSS}} a_t$ is a household's real wealth at time t. If $V_t(a, z)$ is a household's value function at time t given their asset position a at steady state bond prices and labor productivity z, the household problem is

$$
\begin{aligned}
V_0(a_0, z_0) = \max_{\{c_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{h_t(a,z)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] dt \\
\text{s.t. } \frac{q_t}{q_{NSS}} \frac{da_t}{dt} + \frac{dq_t}{dt} \frac{1}{q_{NSS}} a_t = (1-\tau) w_t z_t h_t(a,z) + r_t \frac{q_t}{q_{NSS}} a_t + M_t(z_t; \zeta_t) - c_t \\
d \log(z_t) = -\theta_z \log(z_t) dt + \sigma_z dW_{t,z} \\
a_t \geq 0.
\end{aligned}
$$

Here, W_t is a classical Weiner process (Brownian motion), such that log labor income follows an Ornstein-Uhlenbeck process in the non-stochastic steady state that reverts to the mean at a rate of θ_z. Note that agents do not have bonds in their utility function – but they do value bonds as a means to smooth consumption, particularly in the face of idiosyncratic shocks to their income and a borrowing limit that prohibits their assets from becoming negative. The left-hand side of the consumer's budget constraint represents the value of new assets purchased plus the capital gain associated with existing assets held relative to their steady state values. The right-hand side represents income plus returns net of consumption (savings plus real returns inclusive of capital gains). Like the TANK households outlined in the previous section, the agents can receive transfers from the government $M_t(z_t, \zeta_t)$ that depend on where they are in the joint distribution of assets and incomes. However, I restrict the transfers to be contingent upon household income, not assets.

The household's problem can be recursively formulated as a Hamilton Jacobi Bellman (HJB) equation:

$$
\begin{aligned}
\rho V_t(a,z) = \max_c \Bigg\{ & \left[\frac{c^{1-\gamma}}{1-\gamma} - \frac{h_t(a,z)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \\
& + \frac{\partial V_t}{\partial a}(a,z) \frac{q_{NSS}}{q_t} \left[(1-\tau) w_t z h_t(a,z) + M_t(z_t; \zeta_t) - c + \left(r_t - \frac{dq_t}{dt} \frac{1}{q_t} \right) \frac{q_t}{q_{NSS}} a \right] \\
& + \frac{\partial V_t}{\partial z}(a,z) z \left[\frac{1}{2} \sigma_z^2 - \theta_z \log(z) \right] + \frac{\partial^2 V_t}{\partial z^2}(a,z) \frac{1}{2} \sigma_z^2 z^2 + \frac{\partial V_t}{\partial t}(a,z) \Bigg\}.
\end{aligned} \tag{7}
$$

where households take the path of prices w, r, and q as given, and subsumed into the time subscript of the value functions.

The distribution of households over idiosyncratic states is $\mu_t(a, z)$; it evolves according to the standard Kolmogorov Forward Equation (KFE)

$$\frac{\partial \mu_t}{\partial t}(a,z) = -\frac{\partial}{\partial a}\left(\frac{da_t}{dt}\mu_t(a,z)\right) - \frac{\partial}{\partial z}\left(\frac{\mathbb{E}_t[dz_t]}{dt}\mu_t(a,z)\right) + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left(\sigma^2 z^2 \mu_t(a,z)\right) \tag{8}$$

4.2. Firms and Price Setting

Labor is the only production input in the model economy, such that

$$Y_t = L_t, \tag{9}$$

where Y_t is aggregate real output and L_t is the aggregate number of effective hours worked. Final goods firms are perfectly competitive and face no friction in how they set prices to maximize profits, making wage inflation equal to the final consumption goods' inflation.

Output and employment are demand-determined due to nominal rigidities in the labor market, which are in the style of the decentralized labor union environment of Auclert et al. (2018), which is in turn a modification of Hagedorn et al. (2019) (an earlier adopter of sticky wages in a HANK setting) and Schmitt-Grohé and Uribe (2005). A continuum of decentralized unions hires labor from households and resells it to firms, who differentiate the unions with a constant elasticity of substitution ε_L. Labor supply is demand-determined so that all households work the same number of hours ($h_t(a,z) = L_t/Z$ where $Z = \int\int z\mu(a,z)da\, dz$), and unions are subject to Rotemberg (1982) nominal wage pricing frictions. The result is a nominal forward-looking wage Phillips Curve, which is also the overall Phillips Curve in the economy:

$$\frac{\mathbb{E}_t[d\pi_t]}{dt} = r_t\pi_t - \frac{\varepsilon_\ell}{\theta_w}\frac{L_t}{Z}\int\int\left(h_t(a,z)^{\frac{1}{\eta}} - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)zw_t c_t(a,z)^{-\gamma}\right)da\; dz \tag{10}$$

Inflation today is related to both expected inflation and the average cross-sectional wedge between the disutility of labor and the utility of working for wages, marked up because the unions internalize the effect of supplying more labor on their wage rate.

4.3. Fiscal Policy

The model's fiscal authority collects aggregate taxes (net of transfers) equal to T_t; real government expenditures G_t are included in the following equations for generality but are set to be zero in equilibrium. The aggregate price level in the economy is p_t.

The government borrows using long-term nominal bonds as in Cochrane (2018) and Sims (2002).

When the nominal interest rate is held constant by the central bank, as it is in my baseline specification, the maturity structure of the debt is irrelevant, as nominal bond prices are then constant as well. However, I compare this baseline to one in which monetary policy is active and the policy rate is responsive to macroeconomic variables – in which case the maturity structure of the government debt becomes affects the impulse response functions. As such, the government issues nominal perpetuities $\tilde{B}_t$ at a nominal price of q_t, which pay out exponentially declining coupon payments of $\omega e^{-\omega t}$ per increment of time. As such, ω determines the overall maturity of the government's debt portfolio;[2] as $\omega \to \infty$, government debt becomes instantaneously short-term and must be rolled over immediately with new bonds (analogous to the continuous-time equivalent of a one-period bond in discrete time), while as $\omega \to 0$, each new bond issued becomes a perpetuity.

The market value of real debt outstanding is

$$B_t \equiv \frac{q_t \tilde{B}_t}{p_t}$$

and, as shown in Appendix D.1 and in Cochrane (2018), evolves according to backward-looking equation

$$dB_t = -(T_t - G_t)dt + B_t \left[i_t - \pi_t\right] dt + \frac{d\delta_{q,t}}{q_t} B_t. \tag{11}$$

Here, $d\delta_{q,t} = dq_t - \mathbb{E}_t[dq_t]$ denotes the endogenous expectation error on the nominal price of government debt. Nominal bond prices themselves evolve according to the forward-looking equation

$$\frac{\mathbb{E}_t[dq_t]}{dt} = q_t \left(i_t + \omega - \frac{\omega}{q_t} \right) \tag{12}$$

Notably, since the bonds offer nominal payments, the path of nominal interest rates (and the bond portfolio's maturity structure) determines the evolution of nominal bond prices. In my baseline experiment nominal interest rates are constant, and so bond prices are constant as well. If nominal interest rates respond to inflation via a Taylor rule, as is the case when monetary policy is active, then bond prices move as well.

[2] While this may seem like an arbitrary structure, it can be rationalized by having the government issue debt to maintain an exponentially distributed maturity structure, as shown in Cochrane (2018). From there, one can imagine that government debt is bought by a mutual fund, whose shares are, in turn, owned by households as assets, such that every household effectively owns a representative share of the government's debt portfolio. The details of this are relegated to Appendix D.

4.3.1. Taxes

As a baseline, the fiscal authority in the model taxes labor income at a rate of τ, such that if total effective labor employment in the economy is L_t and real wages are w_t, total income taxes are $\tau w_t L_t$ per unit of time. Households also receive lump-sum transfers from the government, which aggregate to total lump-sum transfers M_t – such that total tax revenue is

$$T_t = \tau w_t L_t - M_t. \tag{13}$$

In the nonstochastic steady state (NSS), the government balances its budget and rebates transfers uniformly such that

$$M_{NSS} = \tau w L_{NSS} - r_{NSS} B_{NSS}$$

Outside of the steady state, transfers can either be made to those below median z (denoted $z_{0.50}$), above median z, or to all households:

$$\begin{aligned} M_t(z, \zeta_t) =& 4Y_{NSS} \times \left(\zeta_{\text{ALL},t} + \frac{1}{0.5} \mathbf{1}\{z \leq z_{0.50}\} \zeta_{\text{BELOW},t} + \frac{1}{0.5} \mathbf{1}\{z > z_{0.50}\} \zeta_{\text{ABOVE},t} \right) \\ & - \kappa_B \left(\frac{q_{NSS}}{q_t} B_t - B_{NSS} \right) \end{aligned} \tag{14}$$

where $\zeta_{ALL,t}, \zeta_{BELOW,t}, \zeta_{ABOVE,t}$ are aggregate shocks that follow (18). The transfer shocks are therefore scaled as a percentage of annual steady state GDP and are also scaled by the mass of the recipients to represent the same amount of aggregate transfer spending.

The last term regulates a fiscal rule that determines whether or not fiscal policy is active or passive. These taxes do not adjust due to revaluations of the government debt through changes in q_t and only respond to changes in debt valued at steady state prices. If $\kappa_B > r_{NSS}$, then taxes automatically adjust to bring debt back to its nonstochastic steady state, making fiscal policy passive. However, if $\kappa_B < r_{NSS}$, then inflation must stabilizes debt, making fiscal policy active. In my baseline scenario, I set $\kappa_B = 0$, rendering fiscal policy unambiguously active.

Total transfers aggregate naturally from their microeconomic counterparts:

$$M_t = \int \int M_t(z, \zeta_t) \mu_t(a, z) da\ dz \tag{15}$$

4.4. Monetary Block

The central bank directly sets nominal interest rates in the economy according to

$$i_t = r^* + \phi_\pi \pi_t + \zeta_{MP,t} \tag{16}$$

where r^* is the interest rate that would prevail in equilibrium in the absence of any aggregate shocks. The active fiscal model can be solved so long as the interest rate rule is "passive," such that $\phi_\pi < 1$. In the baseline specification, I set $\phi_\pi = 0$. In alternative specifications, I consider active monetary policy with $\phi_\pi > 1$.

4.5. Policy Shocks

I assume that aggregate shocks mean-revert at constant rates. As such, they can be written recursively, with the shock of type i at time 0 being given as $\zeta_{i,0}$:

$$d\zeta_{i,t} = -\theta_i \zeta_{i,t} dt \tag{17}$$

or solved forward as a sequence

$$\zeta_{i,t} = e^{-\theta_i t} \zeta_{i,0}. \tag{18}$$

Monetary policy shocks revert at a rate of θ_{MP}, while all fiscal shocks revert at a common rate of θ_{Tax}.

4.6. Market Clearing

Aggregate consumption

$$C_t = \int \int c_t(a, z) \mu_t(a, z) da\ dz \tag{19}$$

is equal to aggregate output:

$$Y_t = C_t. \tag{20}$$

Total hours worked are uniform across households:

$$h_t(a, z) = L_t / Z. \tag{21}$$

The asset market clears when net private wealth equal to aggregate government debt:

$$\frac{q_t}{q_{NSS}} \int \int a \mu_t(a, z) da\ dz = B_t. \tag{22}$$

4.7. HANK Equilibrium

An equilibrium given a sequence of aggregate shocks $(\zeta_t)_{t\geq 0}$, an initial wealth and income distribution $\mu_0(a,z)$, and an initial debt level B_0 is therefore a collection of sequences of macroeconomic aggregates

$$(C_t, L_t, Y_t, B_t)_{t\geq 0}$$

and household-level variables and prices

$$(c_t(a,z), h_t(a,z), M_t(z,\zeta_t), w_t, r_t, i_t, \pi_t, q_t)_{t\geq 0}$$

where

i. saver consumption choices $(c_t(a,z))_{t\geq 0}$ solve (7) given prices and aggregates

ii. labor allocations $(h_{1,t})$ are consistent with the union rule (21)

iii. inflation π_t is consistent with the unions' maximization problem and resulting wage Phillips Curve (10)

iv. nominal government bond prices $(q_t)_{t\geq 0}$ are consistent with (12)

such that

1. Macro aggregates $(Y_t, C_t)_{t\geq 0}$ are consistent with production (9) and aggregation (19)
2. real wages w_t are constant and real rates r_t obey the Fisher equation $r_t = i_t - \pi_t$
3. nominal interest rates $(i_t)_{t\geq 0}$ follow the central bank's policy rule (16)
4. Government taxes and transfers across the population and over time $(M_t(z,\zeta_t))_{t\geq 0}$ follow the rule (14) and aggregate to M_t and T_t via (15) and (13)
5. Government debt B_t given taxes T_t and real rates r_t evolves according to (11)
6. The asset market clears, as in (22). By Walras' law, this also implies goods market clearing (20).

5. Calibration

I calibrate my model largely with parameters that are standard in the HANK literature; they are displayed in Table 1. As in McKay et al. (2016), I calibrate the continuous time income process parameters (θ_z, σ_z^2) via simulated method of moments to match the Floden and Lindé (2001) estimates of the permanent component of annual wage autocorrelation and autoregression variance, residualized for age, occupation, education, and other covariates. I similarly calibrate the time discounting parameter ρ to match a real interest rate of 0.5% quarterly, or roughly 2% annually. Real government debt outstanding is set to 67% of annual GDP in the steady state, so that households' average contemporaneous annualized MPC out of a transfer roughly matches those reported in Auclert et al. (2018). I solve for the model's non-stochastic steady state using the methods outlined in Achdou et al. (2021); select moments from this distribution are reported in Table 2.

The slope of the Phillips Curve is also reported in terms of the coefficient describing the passthrough from marginal labor disutility to prices $\frac{\varepsilon_L}{\theta_\pi} h_t v'(h_t)$, where v is the households' labor disutility, to be comparable with the parameters used in Auclert et al. (2018). In Appendix E.1, I simulate the model with different slopes of the Phillips Curve to evaluate the robustness of my findings to this key parameter. Increasing nominal rigidities predictably amplifies the real effects of active fiscal expansion and smooths the transition of prices, while decreasing nominal rigidities does the opposite. Even so, changing the degree of nominal rigidity in the economy leaves the long-term price level dynamics essentially unchanged, nor does it significantly alter the ordering of sacrifice ratios among the different transfer policies.

The marginal distributions of households along assets and incomes are displayed in Figure 1. Since the distribution of assets contains an atom at the borrowing constraint, I display the cumulative stationary distribution of assets, followed by the probability density of household incomes. The third plot in Figure 1 depicts the aggregate intertemporal MPCs of households in the non-stochastic steady state in response to a year-long transfer that integrates to 1. The iMPCs are aggregated to the annual level to make them comparable with Figures 1 and 2 of Auclert et al. (2018). Households in my model spend roughly 43% of the value of their initial transfer income in the first year when they receive it, 12% a year later, 9% two years later, 7% a year after that, and so on. These iMPCs are roughly consistent with the lower bound presented in Auclert et al. (2018), which uses data from the Italian Survey of Income and Wealth. The plot's dashed lines indicate households' aggregate propensity to spend when a transfer is announced 3 and 7 years in

Table 1: General HANK Model Parameters

Parameter	Symbol	Value	Source or Target
Households			
Internally Calibrated:			
Quarterly Time Discounting	ρ	0.021	$r = 2\%$ Annually
Idiosyncratic Income Shock Variance	σ_z^2	0.017	Floden and Lindé (2001)
Idiosyncratic Shock Mean Reversion	θ_z	0.034	Floden and Lindé (2001)
Assumed from Literature:			
Relative Risk Aversion	γ	2.0	McKay et al (2016)
Frisch Elasticity of Labor	η	0.5	Chetty (2012)
Labor Market			
Labor Elasticity of Substitution	ε_L	10	Philips Curve slope of 0.07
Rotemberg wage adjustment cost	θ_w	100	Philips Curve slope of 0.07
Government			
steady state government debt	B_{NSS}	2.63	HANK $iMPC_0 \approx 0.40$
Geometric maturity structure of debt	ω	0.043	Avg. maturity of 70 months
Income Tax Rate	τ	0.25	
Shocks			
Mean reversion of fiscal shocks	θ_{Tax}	1.0	
Mean reversion of fiscal shocks	θ_{MP}	0.175	Half life of 4 quarters

Table 2: HANK Non-Stochastic steady state Statistics

Description	Symbol	Value
Contemporaneous iMPC (Annual)		0.43
Debt to Annual Income	$B_{NSS}/(4Y_{NSS})$	0.67
Correlation btw. Income and Assets	$\text{Corr}(a, z)$	0.56
Share of households with $a = 0$	$\int \mu_{NSS}(0, z)dz$	0.27
Asset Gini Coefficient		0.75
Income Gini Coefficient		0.31

advance; the tent-shaped spending patterns are again reminiscent of Auclert et al. (2018).

The final plot in Figure 1 depicts the cross-sectional distribution of households' marginal propensities to consume over 4 quarters out of a change to their liquid wealth, calculated using the Feynman-Kac approach employed in Kaplan and Violante (2018). The average roughly matches the first instantaneous iMPC to a contemporaneous shock reported in the previous graph. As one might expect, most agents with no liquid assets and low income have an MPC of close to 1. This MPC rapidly declines as household wealth increases, or (once wage income becomes high enough) as wage income increases.

I assume monetary policy shocks have a half-life of 4 quarters. In contrast, the mean reversion of fiscal shocks is made to be much stronger with $\theta_{\text{Tax}} = 1.0$. This is intended to better replicate the speed with which stimulus checks may be sent out; after 4 quarters, the fiscal shocks almost entirely

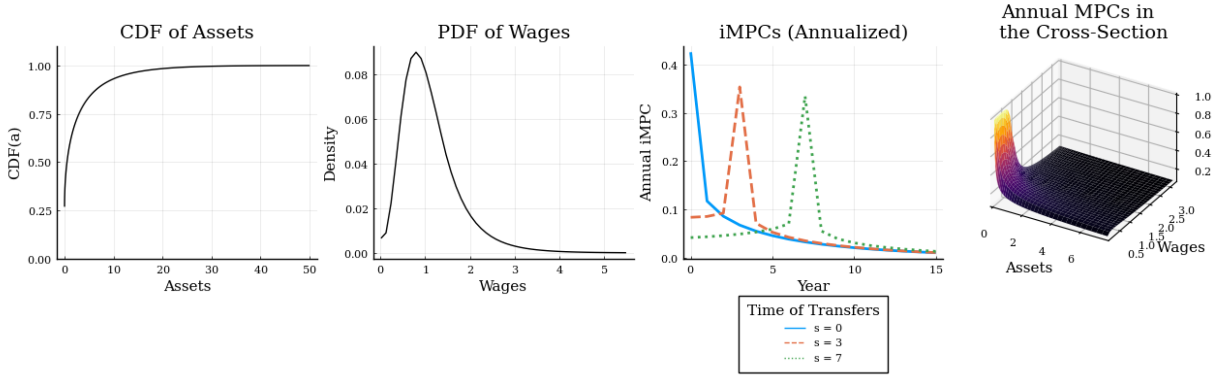


Figure 1: Marginal distributions and marginal propensities to consume in the non-stochastic steady state (both intertemporally and in the cross-section). Assets refer to agents' liquid wealth position a, while wages refers to agents' position in the skill distribution z.

Table 3: Model Parameters (by Policy Regime and Model Type)

	Symbol	AF/PM	PF/PM	PF/AM
Auto Fiscal Adj.	κ	0.0	0.01	0.01
Taylor Rule Coef.	ϕ_π	0.0	0.0	1.05

dissipate. Since the path of the shock in the absence of further perturbations may be described with equation (18), this also means that the cumulative effect of an initial shock of $\zeta_0^{\text{Tax}} = 0.01$ has the interpretation of a 1%-of-annual-GDP disbursal of lump-sum stimulus checks.[3]

To choose the fiscal and monetary policy regime, I set the parameters in 3 depending on the setting. The active fiscal/passive monetary (AF/PM) policy mix is attained by turning automatic fiscal adjustments off ($\kappa = 0$) and making nominal rates constant ($\phi_\pi = 0$). For the passive fiscal/passive monetary (PF/PM) regime, I set $\kappa = 0.01$; for the passive fiscal/active monetary (PF/AM) setting, I additionally make monetary policy active by setting $\phi_\pi = 1.05$.

6. HANK Results

I examine the cumulative effect of shocks on the model economy. As such, I construct $\mathcal{C}Y_t$, the accumulated increase in GDP relative to the non-stochastic steady state, as

$$\mathcal{C}Y_s \equiv \frac{1}{Y_{NSS}} \int_0^t (Y_s - Y_{NSS}) ds. \tag{23}$$

[3]For example, if the United States economy in 2019 were to be taken to be the non-stochastic steady state, this would be a spending program of \$210 billion.

Cumulative inflation $\mathcal{C}\pi_t$, the total increase in the price level following the shock, can be found by solving the differential equation $\frac{dp_t}{dt} = \pi_t p_t$ forward in time with the initial price level as given:

$$1 + \mathcal{C}\pi_t = \exp\left(\int_0^t \pi_s ds\right).$$

A cumulative sacrifice ratio, the accumulated trade-off as of time t between *annual* real GDP and the change in the price level in response to a shock, may be inferred as $(\mathcal{C}Y_t/4)/\mathcal{C}\pi_t$.

I report these accumulated quantities at different time horizons for the active fiscal/passive monetary setting in Table 4. The first row details the cumulative sum of the output gaps as a percent of annual steady state GDP, respectively accumulated up to 1 year and up to 50 quarters, for transfers to all, below-median income, and above-median income households. Since the transfers are almost entirely paid out after four quarters and accumulate to 1% of annual GDP, this row could also be read as the fiscal transfer multiplier of the different policy shocks. The total rise in the price level for the different transfers and time horizons is reported in the next line. Finally, I report the cumulative sacrifice ratio (the ratio of the first and second lines) in the last row.

Transfers to low income households boost cumulative output gaps by more than twice as much as transfers to high income households and 33% more than untargeted transfers in the first year. After 50 quarters, the amount declines slightly as output overshoots – but transfers to low income households still generate a 59% and 24% larger accumulation of real output gaps than high income transfers and untargeted transfers, respectively. This is despite the fact that the 50-quarter rise in the price level is nearly the same for both untargeted transfers and transfers to low-income households, and actually 9% lower than the rise in the price level associated with transfers to the high income (although in the first year, targeted transfers to the low income do yield significantly more inflation). Both in the short-term and in the longer-term, the cumulative sacrifice ratios related to reducing net transfers to the high income households are substantially lower than reducing transfers to low income households.

The full paths of aggregate cumulative output and inflation in the HANK model following shocks in active fiscal/passive monetary, passive fiscal/passive monetary, and passive fiscal/active monetary environments are displayed in Figure 2; the unaccumulated impulse response functions used to create the graph are displayed in Figure 3. Although there are more pronounced short-term differences over the first few years, the solid red lines – which depict the rise in the price level in the active fiscal/passive monetary experiment – all settle to similar levels over time in the first three

	Transfers to **All**		Transfers to **Low-Income**		Transfers to **High-Income**	
	1 yr	50 qtrs	1 yr	50 qtrs	1 yr	50 qtrs
$\mathcal{C}Y_t/4$	0.66%	0.59%	0.90%	0.73%	0.43%	0.46%
$\mathcal{C}\pi_t$	1.58%	1.47%	1.85%	1.40%	1.34%	1.54%
Sac. Ratio	0.42	0.40	0.49	0.52	0.32	0.30

Table 4: Cumulative annualized output gaps ($\mathcal{C}Y_t/4$), inflation ($\mathcal{C}\pi_t$), and sacrifice ratios for fiscal transfers to different groups in the active fiscal/passive monetary HANK model.

graphs depicting a response to a fiscal shock, as stipulated in Table 4. The solid blue lines, which depict the cumulative quarterly output gap as a percent of quarterly GDP, in contrast, settle on different levels depending on where the transfer payments went.

Strikingly, when both fiscal policy and monetary policy are passive, the impulse response functions (displayed with dotted lines) are nearly unchanged from their counterparts when only fiscal policy is passive. This further supports the stipulation of Hagedorn (2024) that active fiscal policy is *not* selecting the equilibrium in incomplete markets models. In contrast, when monetary policy is active, active monetary policy selects a markedly different equilibrium.

Although the model features endogenous MPC and wealth distributions, income inequality, and precautionary savings motives, the impulse response functions of the HANK model display the same qualitative patterns as the TANK ones in Appendix C. When transfers are sent out to low-income agents with fewer assets and higher MPCs, the output response is larger; the inflation response is largely the same regardless of the distribution of the recipients. The first two rows of plots in Figure 3 shows that although inflation and output spikes higher at the moment of the shock when transfers are given to poorer households, the inflation response is less persistent than when the transfer is made to lower-MPC agents, in keeping with the intuition developed in earlier sections.

What drives the expansion of real output in the active-fiscal/passive-monetary HANK model after transfer payments go out? Perhaps unsurprisingly, the majority of the response is driven by the increase in households' aggregate demand following an increase in their transfer income net of taxes – particularly because the persistence of the transfers is low. In Figure 4, I decompose the output impulse response function into a component associated with the households' response to the transfers themselves (in yellow), the path of real interest rates r (in red), and changes in the aggregate demand for labor L (in blue). The paths of each of these inputs, determined in equilibrium, are taken as given by households; the colored regions of the plot depict how each contributes to the total movement of real GDP, which is depicted in the black dashed line. While

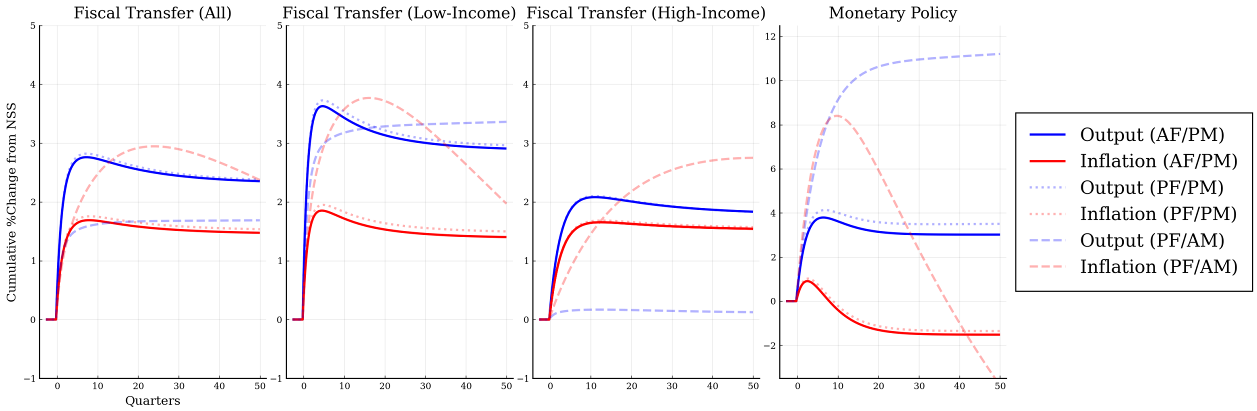


Figure 2: Cumulative impulse response functions in a HANK model. Shocks 1% of annual GDP increases in transfers to all agents, below-median income agents, and above-median income agents, respectively, along with 1% nominal interest rate cuts.

an increase in employment and labor income does contribute to the expansion (and the increase in aggregate equilibrium labor is what produces the goods that households consume), the rise in aggregate consumption is predominantly driven by the increase in net transfer income that high-MPC households receive (the policy's direct effect). When low-MPC households receive the checks, general equilibrium effects play a larger role in the smaller real GDP response; households are motivated to spend following the decline in real rates following inflation, and then to save again to rebuild their precautionary savings following the boom once real rates of return have recovered.

To contextualize the magnitudes of the responses, it is beneficial to compare the magnitudes of the economy's response to an active fiscal shock with that of a shock more commonly studied in the macroeconomic literature: an interest rate shock in a passive fiscal/active monetary setting. The last graph in 2 and the last column of 3 depict the model's response to a 1% reduction in the central bank's policy rate rule (16). An expansionary monetary policy shock in an active monetary setting is much more powerful overall than the transient active fiscal stimulus. The reader should note that these last plots feature a different y-axis than those of the fiscal experiments. As a reminder of the calibration, the monetary shock is much more persistent than the fiscal shock and has a half-life of 4 quarters (emblematic of the persistence of monetary policy) while the fiscal shock has a half-life of about 8 months (to simulate the quick disbursal of stimulus checks).

Following the active monetary policy shock, output rises by nearly 1.4% of annual steady state GDP in the first year (6% of quarterly); the output gaps accumulate to over 2.8% of steady state annual GDP (11% of quarterly) over the plotted 50-quarter horizon. By contrast, even an active fiscal targeted transfer to low-income households boosts annual GDP by 0.9% in the first year (3.6%

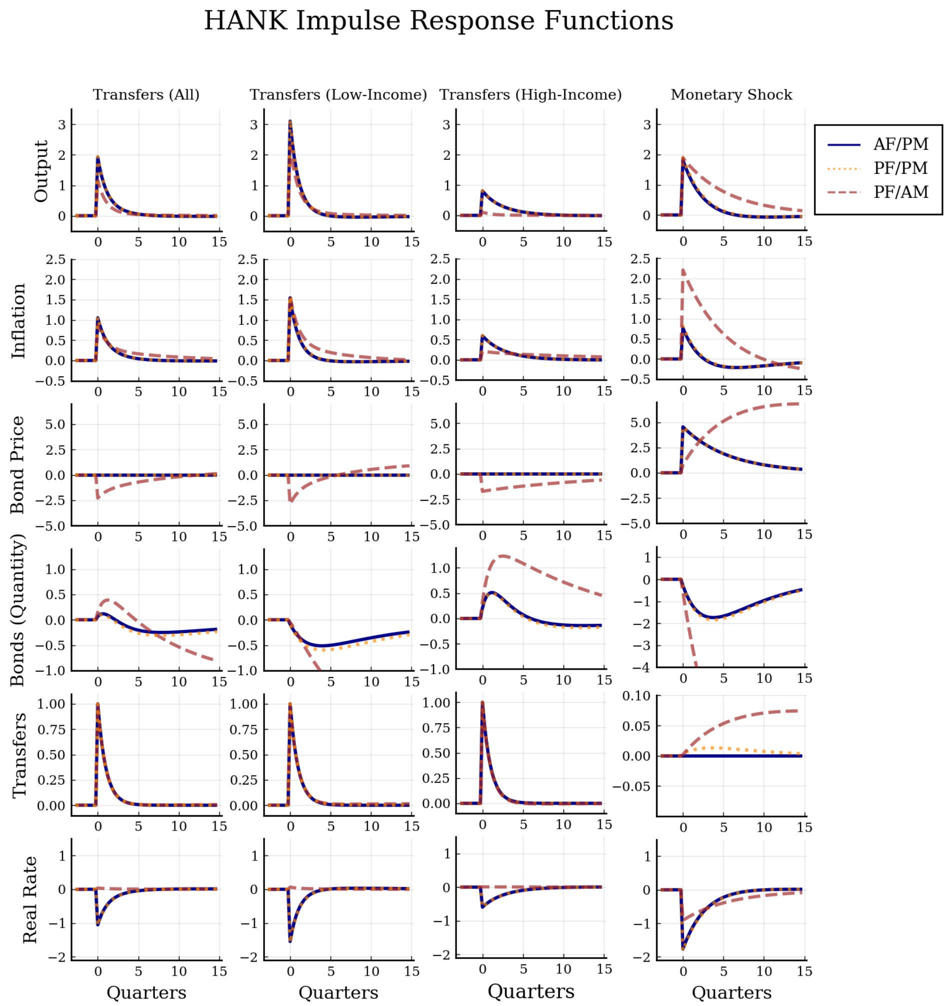


Figure 3: Impulse response functions (unaccumulated) to policy shocks in the HANK environment. Solid lines correspond to the HANK economy under an active-fiscal/passive monetary policy regime. Dashed lines refer to the economy under a slow fiscal adjustment passive-fiscal/active monetary one, as parameterized by Panel B of Table 3. Dotted lines correspond to a passive-fiscal/passive monetary policy mix. All variables are presented as deviations from their quarterly values in the non-stochastic steady state except for transfers, which are reported as a percentage of annual real GDP in the non-stochastic steady state.

in quarterly terms), and only 0.7% of annual GDP (2.9% of quarterly) by the end of 50 quarters. As such, active monetary policy generates an expansion that is nearly four times larger than that of even targeted active fiscal policy. In the US context using 2019 numbers, this would imply that lowering the Fed's interest rate target by 100 basis points and gradually returning to the the older target over two years stimulates the economy by many times more than sending stimulus checks of 1% of GDP (roughly $200 billion) to below-median income households. In Appendix C, I show that these differences in relative magnitudes are not just a feature of the more sophisticated HANK model; the simpler two-agent framework of the last section delivers them as well.

As shown in the last panel of Figure 2, the relative magnitudes change substantially if the

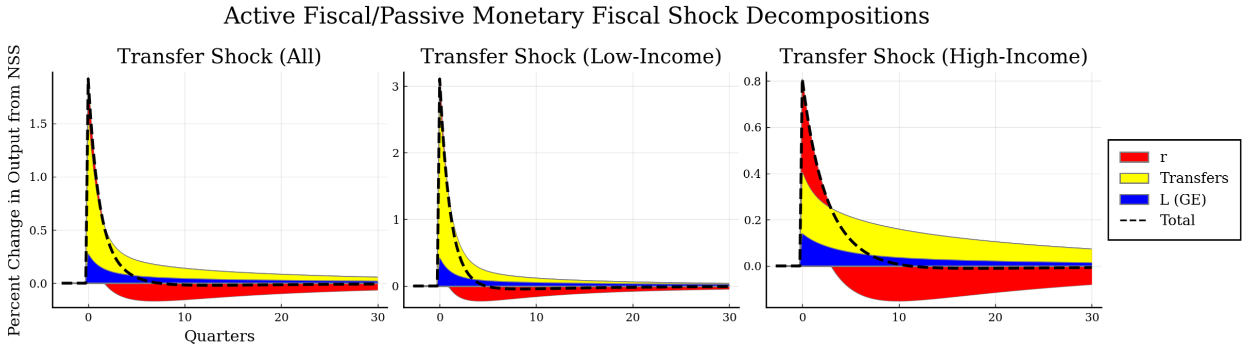


Figure 4: Decompositions of the real output impulse response function in an active fiscal/passive monetary HANK. Each channel represents the heterogeneous agents' response to i) real interest rates and bond prices (in red), ii) transfers (in yellow), and general equilibrium changes in labor demand (in blue). The colored regions add up to the dashed black line.

monetary policy shock takes place while fiscal policy is active and monetary policy is passive. The shock acts then through the channels described in Cochrane (2018): a persistent reduction in the path of nominal interest rates boosts nominal bond prices while the price level takes time to adjust, resulting in a real asset price appreciation. Asset holders feel wealthier, while agents who might have otherwise been marginal asset buyers respond to the high asset prices by shifting their spending from assets to goods. Both effects increase aggregate demand in the goods market, driving up the prices of goods and lowering the rate of return of assets such that the markets for both clear. Through this active fiscal monetary channel, the boost to real output is then only 0.8% of annual steady state GDP in the first year and 0.75% after 50 quarters. Rather than being several times more powerful, monetary policy becomes slightly less powerful in boosting GDP than a targeted active fiscal expansion in the short-term and about as powerful in the long-term, and slightly more powerful than an untargeted active fiscal expansion, where annual GDP rises by 0.66% in the first year and accumulates to 0.59% of steady state GDP after 50 quarters.

To explain the differences, I again decompose the real GDP impulse response functions to a nominal rate cut into three components: households' response to changes in real interest rates (now the direct effect of the policy), changes in labor demand, and changes in transfer income. I consider the shock in active fiscal/passive monetary, passive fiscal/passive monetary, and passive fiscal/active monetary settings. The resulting decomposition appears in Figure 5.

First, a rate cut in an active fiscal/passive monetary environment acts primarily (85% on impact) direct channels, with the remainder attributed to the change in general equilibrium labor demand. The household does not respond to a change in transfers, since in the absence of passive budget-balancing adjustments transfers do not change in response to the interest rate reduction.

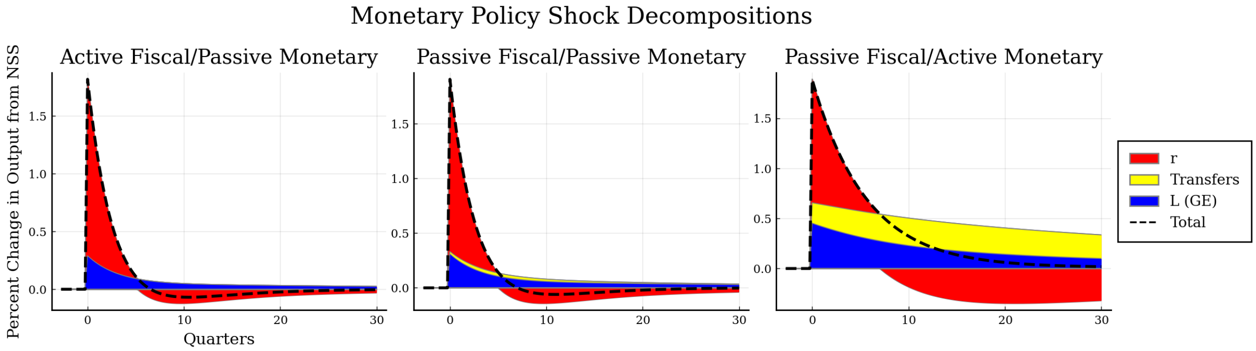


Figure 5: Decomposition of the impulse response of real GDP to a 1% nominal rate cut shock, in active fiscal/passive monetary, passive monetary/passive fiscal, and passive fiscal/active monetary settings.

If monetary policy is kept passive but fiscal policy is made passive as well, then the government gradually adjusts transfers to balance the budget. As such, the government cuts taxes and increases transfers uniformly to all households in response to lower real interest rates, which lower real interest expenses and provide the government with more fiscal space. However, the impulse response function is almost identical to the preceding active fiscal/passive monetary ones. Although automatic fiscal adjustments stabilize the debt, the speed of fiscal adjustment is slow. In both simulations, lower nominal rates eventually pull down inflation through a neo-Fisherian effect and cause real rates to quickly return to baseline levels, as shown in Figure 3. As such, in a setting where both types of policy are passive, the amount of new spending that the government can do thanks to the lower rates is small and negligibly affects the path of macroeconomic aggregates.

The final panel of Figure 5 makes the source of monetary policy's lasting and powerful impact in a passive fiscal/active monetary environment clear. In addition to the direct effect of real interest rates and the general equilibrium effect of higher labor employment, the latter policy mix generates larger automatic increases in transfers in response to the rate cut, to which households respond strongly. The reason for this is not simply because fiscal policy is passive; if passive fiscal policy was the only prerequisite, the passive fiscal/passive monetary response would be similarly strong. Rather, Figure 3 indicates that the passive increase in transfers is much larger in the passive fiscal/active monetary environment; the automatic transfers are only about 10% the size of the active fiscal policy shocks at their peak, but they last much longer and additionally provide precautionary insurance to non-hand-to-mouth households, further stimulating aggregate demand.

These transfers can be larger for longer because real interest rates stay depressed for longer in the active monetary environment, which rapidly lowers aggregate real debt to accommodate more spending. Passive monetary policy brings real rates to their neutral steady state values quickly via the neo-Fisherian effect. However, if the central bank responds to inflation by raising real rates, then a cut to the nominal rate target leads to a jump in inflation. The central bank then responds to this by raising nominal rates – leading nominal rates to fall by less than the value of the monetary policy shock. As such, the active central bank reduces the neo-Fisherian pull of reduced interest rates on inflation, diminishing the degree to which real rates fall immediately but increasing the length of time in which real interest rates are low.

7. Discussion

Because low-income households have low liquid wealth and high marginal propensities to consume, sending deficit-financed transfers to them leads to a sharp boost in output. However, if the central bank does not raise nominal interest rates in response to inflation, then the distribution of transfer recipients has little impact on how much inflation transpires. Inflation accumulates until the nominal assets issued by the government and held by households have returned to steady state levels, regardless of who received the funds; cumulative inflation is not sensitive to heterogeneity in MPCs.

Transfers to the low-income thus generate larger amounts of GDP relative to the amount of inflation they produced, compared to when the checks go to wealthier high-income segments of the population. This is consistent with the baseline Phillips Curve; when output rises quickly, firms take time to adjust their prices and respond to future expected output gaps, not previous ones, leading the overall rise in the price level to trail a sharp rise in output. Conversely, this dynamic has strong implications for "sacrifice ratios": abating inflation by cutting transfers to the low-income depresses real GDP by much more than similar inflation abatement accomplished by lump-sum tax increases on the rich, as sacrifice ratios themselves are positively related to the speed with which the output gaps occur.

The strength of fiscal policy relative to monetary policy depends strongly on the policy mix adopted by institutions in the model economy. If central banks do not raise real interest rates in response to inflation, then the equilibria are similar whether or not fiscal policy is active or passive, as neo-Fisherian effects restore real rates to steady state levels quickly. The stimulus effect of low rates is then similar to that of fiscal policy tools. However, if central banks respond with active monetary policy, as is standard in New Keynesian models, then the passive-fiscal equilibrium generates persistent automatic transfers that stimulate the economy far beyond what even active fiscal transfers to high-MPC agents can deliver.

The intuition that the price level might strongly depend on how some households behave more like "savers" or "spenders" after receiving their checks is also not quantitatively borne out in a HANK model. As Auclert et al. (2018) notes, optimizing agents will eventually want to spend the present value of whatever they receive, such that the present value of iMPCs aggregates to one, even if they smooth that consumption spending over time. Eventually, for the asset market to clear and for the economy to return to its non-stochastic steady state, inflation occurs to bring nominal private assets back to stable real levels.

When this is the case, one can predict the long-term inflationary impact of a policy without much knowledge of its distributional consequences or implications for employment and output. But is this the case? Less conventional, but perhaps important, theoretical complications could emerge if models contain behavioral agents with MPCs that are truly zero, such as in Auclert et al. (2023b), leading them to act as a permanent real asset sink. Inflation might play a less predictable, and perhaps reduced, role in the equilibrium dynamics of such models. Empirically, there also appears to be an opening for more work examining how inflation does or does not ensue when governments do not have a credible plan to pay down their debt through conventional means following unexpected deficit spending. Ultimately, recent theories of the price level and models with meaningful heterogeneity open up new ways to understand how fiscal and monetary policy interact to influence macroeconomic aggregates – potentially with profound implications for policy in the real world.

References

Acharya, S., Dogra, K., 2020. Understanding hank: Insights from a prank. Econometrica 88, 1113–1158. URL: https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA16409, doi:https://doi.org/10.3982/ECTA16409, arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA

Achdou, Y., Han, J., Lasry, J.M., Lions, P.L., Moll, B., 2021. Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach. The Review of Economic Studies 89, 45–86. URL: https://doi.org/10.1093/restud/rdab002, doi:10.1093/restud/rdab002, arXiv:https://academic.oup.com/restud/article-pdf/89/1/45/42137446/rdab002.pdf.

Ahn, S., Kaplan, G., Moll, B., Winberry, T., Wolf, C., 2018. When inequality matters for macro and macro matters for inequality. NBER Macroeconomics Annual 32, 1–75. URL: https://doi.org/10.1086/696046, doi:10.1086/696046, arXiv:https://doi.org/10.1086/696046.

Auclert, A., 2018. Discussion of "The Fiscal Multiplier" by Marcus Hagedorn, Iourii Manovskii and Kurt Mitman. Economic Fluctuations and Growth Meeting. Presented at the San Francisco Federal Reserve.

Auclert, A., Bardóczy, B., Rognlie, M., Straub, L., 2021. Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. Econometrica 89, 2375–2408. URL: https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA17434, doi:https://doi.org/10.3982/ECTA17434, arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA

Auclert, A., Rognlie, M., Straub, L., 2018. The Intertemporal Keynesian Cross. Working Paper 25020. National Bureau of Economic Research. URL: http://www.nber.org/papers/w25020, doi:10.3386/w25020.

Auclert, A., Rognlie, M., Straub, L., 2023a. Determinacy and existence in the sequence space. Working Paper .

Auclert, A., Rognlie, M., Straub, L., 2023b. The Trickling Up of Excess Savings. Working Paper 30900. National Bureau of Economic Research. URL: http://www.nber.org/papers/w30900, doi:10.3386/w30900.

Ball, L., 1994. What Determines the Sacrifice Ratio?. The University of Chicago Press. book Monetary Policy, Chapter 5. pp. 155–193. URL: http://www.nber.org/chapters/c8332.

Bayer, C., Luetticke, R., 2020. Solving discrete time heterogeneous agent models with aggregate risk and many idiosyncratic states by perturbation. Quantitative Economics 11, 1253–1288. URL: https://onlinelibrary.wiley.com/doi/abs/10.3982/QE1243, doi:https://doi.org/10.3982/QE1243, arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.3982/QE1243.

Bianchi, F., Faccini, R., Melosi, L., 2023. A Fiscal Theory of Persistent Inflation*. The Quarterly Journal of Economics 138, 2127–2179. URL: https://doi.org/10.1093/qje/qjad027, doi:10.1093/qje/qjad027, arXiv:https://academic.oup.com/qje/article-pdf/138/4/2127/51765358/qjad027.pdf.

Blanchard, O.J., Kahn, C.M., 1980. The solution of linear difference models under rational expectations. Econometrica 48, 1305–1311. URL: http://www.jstor.org/stable/1912186.

Chetty, R., 2012. Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply. Econometrica 80, 969–1018. URL: http://www.jstor.org/stable/41493842.

Cochrane, J.H., 2018. Stepping on a rake: The fiscal theory of monetary policy. European Economic Review 101, 354–375. URL: https://www.sciencedirect.com/science/article/pii/S001429211730199X, doi:https://doi.org/10.1016/j.euroecorev.2017.10.011.

Cochrane, J.H., 2023. The Fiscal Theory of the Price Level. Princeton University Press. URL: http://www.jstor.org/stable/j.ctv2sbm8kh.

Farmer, R., Zabczyk, P., 2019. A Requiem for the Fiscal Theory of the Price Level. IMF Working Papers 2019/219. International Monetary Fund. URL: https://ideas.repec.org/p/imf/imfwpa/2019-219.html.

Floden, M., Lindé, J., 2001. Idiosyncratic risk in the united states and sweden: Is there a role for government insurance? Review of Economic Dynamics 4, 406–437. URL: https://www.sciencedirect.com/science/article/pii/S1094202500901212, doi:https://doi.org/10.1006/redy.2000.0121.

Hagedorn, M., 2016. A Demand Theory of the Price Level. 2016 Meeting Papers 941. Society for Economic Dynamics. URL: https://ideas.repec.org/p/red/sed016/941.html.

Hagedorn, M., 2023. Local determinacy in incomplete-markets models. CEPR Discussion Paper No. 18642. URL: https://cepr.org/publications/dp18642.

Hagedorn, M., 2024. The failed theory of the price level. Working Paper. URL: https://cepr.org/publications/dp18786.

Hagedorn, M., Manovskii, I., Mitman, K., 2019. The Fiscal Multiplier. Working Paper 25571. National Bureau of Economic Research. URL: http://www.nber.org/papers/w25571, doi:10.3386/w25571.

Kaplan, G., Moll, B., Violante, G.L., 2018. Monetary policy according to hank. American Economic Review 108, 697–743. URL: https://www.aeaweb.org/articles?id=10.1257/aer.20160042, doi:10.1257/aer.20160042.

Kaplan, G., Nikolakoudis, G., Violante, G.L., 2023. Price level and inflation dynamics in heterogeneous agent economies. Working paper. .

Kaplan, G., Violante, G.L., 2018. Microeconomic Heterogeneity and Macroeconomic Shocks. Working Paper 24734. National Bureau of Economic Research. URL: http://www.nber.org/papers/w24734, doi:10.3386/w24734.

Leeper, E.M., 1991. Equilibria under 'active' and 'passive' monetary and fiscal policies. Journal of Monetary Economics 27, 129–147. URL: https://www.sciencedirect.com/science/article/pii/030439329190007B, doi:https://doi.org/10.1016/0304-3932(91)90007-B.

McKay, A., Nakamura, E., Steinsson, J., 2016. The power of forward guidance revisited. American Economic Review 106, 3133–58. URL: https://www.aeaweb.org/articles?id=10.1257/aer.20150063, doi:10.1257/aer.20150063.

Onatski, A., 2006. Winding number criterion for existence and uniqueness of equilibrium in linear rational expectations models. Journal of Economic Dynamics and Control 30, 323–345. URL: https://www.sciencedirect.com/science/article/pii/S0165188905000357, doi:https://doi.org/10.1016/j.jedc.2005.02.001.

Reiter, M., 2009. Solving heterogeneous-agent models by projection and perturbation. Journal of Economic Dynamics and Control 33, 649–665. URL:

https://www.sciencedirect.com/science/article/pii/S0165188908001528, doi:https://doi.org/10.1016/j.jedc.2008.08.010.

Rotemberg, J.J., 1982. Sticky prices in the united states. Journal of Political Economy 90, 1187–1211. URL: http://www.jstor.org/stable/1830944.

Schmitt-Grohé, S., Uribe, M., 2005. Optimal fiscal and monetary policy in a medium-scale macroeconomic model. NBER Macroeconomics Annual 20, 383–425. URL: http://www.jstor.org/stable/3585431.

Sims, C.A., 2002. Solving Linear Rational Expectations Models. Computational Economics 20, 1–20. URL: https://ideas.repec.org/a/kap/compec/v20y2002i1-2p1-20.html.

Werning, I., 2015. Incomplete Markets and Aggregate Demand. NBER Working Papers 21448. National Bureau of Economic Research, Inc. URL: https://ideas.repec.org/p/nbr/nberwo/21448.html.

Appendix A. Simple Model Derivations

Appendix A.1. The Phillips Curve and Cumulative Inflation and Output Gaps

Equation (4) can be integrated forward to write

$$\pi_t = \nu \int_t^{\infty} e^{-\rho s} \widehat{Y}_s ds$$

Accumulating inflation from time 0 to a terminal time T, I define $\mathcal{C}\pi_T$ as the rise in the price level by time T and approximate it as

$$\mathcal{C}\pi_T \equiv \exp\left(\int_0^T \pi_t dt\right) - 1 \approx \int_0^T \pi_t dt = \nu \int_0^T \left(\int_t^{\infty} e^{-\rho s} \widehat{Y}_s ds\right) dt$$

The timing of the output gaps matter. The region being integrated over is the triangle defined by $0 \leq t \leq T$ and $t \leq s \leq \infty$. This is the same region as the one bounded by $0 \leq s \leq T$ and $0 \leq t \leq \min(s, T)$. Changing the order of integration,

$$= \nu \int_0^{\infty} \int_0^{\min(s,T)} e^{-\rho s} \widehat{Y}_s dt\ ds = \nu \int_0^T s e^{-\rho s} \widehat{Y}_s ds + \nu \int_T^{\infty} T e^{-\rho s} \widehat{Y}_s ds$$

and taking $T \to \infty$,

$$\int_0^{\infty} \pi_t dt = \nu \int_0^{\infty} t e^{-\rho t} \widehat{Y}_t dt \tag{A.1}$$

Suppose the output gaps jump and decay back to steady state at a rate of λ_Y, such that $\widehat{Y}_t = \lambda e^{-\lambda t} \mathcal{C}Y_{\infty}$, where $\mathcal{C}Y_{\infty} \equiv \int_0^{\infty} \widehat{Y}_t dt$ is the cumulative output gap over time. In that case,

$$\int_0^{\infty} \pi_t dt = \nu \int_0^{\infty} t e^{-\rho t} \lambda e^{-\lambda t} \mathcal{C}Y_{\infty} dt = \nu \lambda \mathcal{C}Y_{\infty} \int_0^{\infty} t e^{-(\rho+\lambda)t} dt$$

such that

$$\mathcal{C}\pi_{\infty} / \mathcal{C}Y_{\infty} = \nu \frac{\lambda}{(\lambda + \rho)^2}$$

If $\rho \approx 0$, then $\mathcal{C}\pi_{\infty}/\mathcal{C}Y_{\infty} \approx \nu/\lambda$. The asymptotic amount of cumulative inflation relative to cumulative output tends to increase with the slope of the Phillips Curve, but *decrease* when output rises faster. More output in a given time increment increases the amount of inflation, but nominal rigidities imply that faster growth in output mean that prices cannot, in a sense keep up. The Phillips Curve is forward looking; previous output gaps are already sunk from the perspective of the firm. If a lot of growth happens quickly and then subsides, that past growth no longer matters for period t inflation; all that matters are future output gaps.

Appendix A.2. Debt Evolution: Inflation and Nominal Debt

Begin with the debt evolution equation:

$$\frac{dB_t}{dt} = -T_t + r_t B_t$$

Solving the ODE forward with an integrating factor of $\int_t^\tau r_s ds$ and assuming the real value of debt does not explode,

$$B_t = \int_t^\infty e^{-\int_t^\tau r_s ds} T_\tau d\tau$$

Dividing by steady state real debt, taxes, and real interest rates as (B, T, r) (no time indexes) and writing $B_t = Be^{\widehat{B}_t}$, $T_t = Te^{\widehat{T}_t}$, $r_t = \widehat{r}_t + r$,

$$e^{\widehat{B}_t} = \int_t^\infty e^{-\int_t^\tau (\widehat{r}_s + r) ds} \frac{T}{B} e^{\widehat{T}_\tau} d\tau$$

Log-linearizing, including writing $\exp\left(-\int_t^\tau \widehat{r}_s ds\right) \approx 1 - \int_t^\tau \widehat{r}_s ds$

$$(1 + \widehat{B}_t) \approx \frac{T}{B} \int_t^\infty e^{-(\tau - t)r} \left(1 - \int_t^\tau \widehat{r}_s ds\right) (1 + \widehat{T}_\tau) d\tau \approx \frac{T}{B} \int_t^\infty e^{-(\tau - t)r} \left(1 - \int_t^\tau \widehat{r}_s ds + \widehat{T}_\tau\right) d\tau$$

where the second approximation follows from the cross-terms of the hatted variables being very small. Note that with a $u = -(\tau - t)r$ substitution, $\frac{T}{B} \int_t^\infty e^{-(\tau - t)r} d\tau = \frac{1}{r} \frac{T}{B} \int_0^{-\infty} e^u du = 1$. As such,

$$\widehat{B}_t \approx \frac{T}{B} \int_t^\infty e^{-(\tau - t)r} \left(\widehat{T}_\tau - \int_t^\tau \widehat{r}_s ds\right) d\tau$$

Note that as $r_t = i_t - \pi_t$, it follows that $\widehat{r}_t = \widehat{i}_t - \widehat{\pi}_t$, where the hatted variables denote deviations from the non-stochastic steady state. If the central bank does not change nominal interest rates, and if prices do not jump on impact such that real debt does not jump on impact,

$$\widehat{B}_t \approx \frac{T}{B} \int_t^\infty e^{-(\tau - t)r} \left(\widehat{T}_\tau - \int_t^\tau (\widehat{i}_s - \widehat{\pi}_s) ds\right) d\tau$$

becomes

$$0 \approx \frac{T}{B} \int_t^\infty e^{-(\tau - t)r} \left(\widehat{T}_\tau + \int_t^\tau \widehat{\pi}_s ds\right) d\tau$$

Such that

$$\int_t^\infty e^{-(\tau-t)r}\left(\int_t^\tau \widehat{\pi}_s ds\right)d\tau = -\int_t^\infty e^{-(\tau-t)r}\widehat{T}_\tau d\tau$$

The region demarcated by $t \leq s \leq \tau$ and $t \leq \tau \leq \infty$ can be equivalently demarcated by $s \leq \tau \leq \infty$ and $t \leq s \leq \infty$. As such,

$$\int_t^\infty e^{-(\tau-t)r}\left(\int_t^\tau \widehat{\pi}_s ds\right)d\tau = \int_t^\infty \int_s^\infty e^{-(\tau-t)r}\widehat{\pi}_s d\tau \; ds = \int_t^\infty \widehat{\pi}_s \left(\int_s^\infty e^{-(\tau-t)r} d\tau\right) ds$$

$$= \frac{1}{r}\int_t^\infty e^{-(s-t)r}\widehat{\pi}_s ds$$

Using the fact that $r = \frac{T}{B}$:

$$\int_t^\infty e^{-(\tau-t)r}\widehat{\pi}_\tau d\tau = -\frac{T}{B}\int_t^\infty e^{-(\tau-t)r}\widehat{T}_\tau d\tau \tag{A.2}$$

Suppose $\mathcal{C}\pi_\tau = (1 - e^{-\lambda_\pi(\tau-t)})\mathcal{C}\pi_\infty$ for $\tau \geq t$. Note that this implies π_t jumps by a factor of $\lambda\mathcal{C}\pi_\infty$ on impact, and mean reverts with an exponential rate of λ, such that $\pi_\tau = \lambda_\pi e^{-\lambda_\pi(\tau-t)}\mathcal{C}\pi_\infty$. Then the present value of the path of inflation is

$$\int_t^\infty e^{-(\tau-t)r}\widehat{\pi}_\tau d\tau = \int_t^\infty e^{-(\tau-t)(r+\lambda_\pi)}\lambda_\pi \mathcal{C}\pi_\infty d\tau = \frac{\lambda_\pi}{r+\lambda_\pi}\mathcal{C}\pi_\infty$$

such that

$$\int_t^\infty \widehat{\pi}_\tau d\tau = -\left(1 + \frac{r}{\lambda}\right)\frac{T}{B}\int_t^\infty e^{-(\tau-t)r}\widehat{T}_\tau d\tau.$$

Note that $r\frac{T}{B} = r^2 \approx 0$ when r is small, so the effect of the timing of the output gaps on cumulative inflation is small if inflation mean reverts with a half life of a few quarters.

Appendix A.3. TANK Euler Equation

I derive the saver household's Euler equation with bonds in the utility function; to obtain the standard Euler equation, I can set $\phi = 0$. The saver household's problem is

$$\begin{aligned}
&\max_{(c_{1,t})_{t\geq 0}} \mathbb{E}\int_0^\infty e^{-\rho t}\left[\frac{c_{1,t}^{1-\gamma}}{1-\gamma} - \frac{h_{1,t}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \psi\frac{a_t^{1-\gamma_b}}{1-\gamma_b}\right] dt \\
&\text{s.t. } \frac{da_t}{dt} = (1-\tau)w_t h_{1,t} + r_t a_t + M_{1,t} - c_t \\
&\lim_{T\to\infty} \mathbb{E}[e^{-\int_0^T r_t dt} a_T] \geq 0
\end{aligned} \tag{A.3}$$

The Hamilton-Jacobi Bellman equation is (suppressing the value function's dependence on aggregate shocks by subsuming them into the time index)

$$\rho V_t(a) = \max_{c_{1,t}}\left\{\left[\frac{c_{1,t}^{1-\gamma}}{1-\gamma} - \frac{h_{1,t}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \psi\frac{a_t^{1-\gamma_b}}{1-\gamma_b}\right] + \frac{\partial V_t(a)}{\partial a}[(1-\tau)w_t h_{1,t} + r_t a_t + M_{1,t} - c_t] + \frac{\mathbb{E}_t[\partial V_t(a)]}{\partial t}\right\}$$

Taking first-order conditions,

$$c_{1,t}^{-\gamma} = \frac{\partial V_t(a)}{\partial a}$$

And with the Envelope Theorem,

$$\begin{aligned}
\rho\frac{\partial V_t(a)}{\partial a} =& \psi a_t^{-\gamma_b} + \frac{\partial}{\partial a}\left(\frac{\partial V_t(a)}{\partial a}[(1-\tau)w_t h_{1,t} + r_t a_t + M_{1,t} - c_t]\right) + \frac{\mathbb{E}_t[\partial(\partial V_t(a)/\partial a)]}{\partial t} \\
=& \psi a_t^{-\gamma_b} + \frac{\partial^2 V_t(a)}{\partial a^2}[(1-\tau)w_t h_{1,t} + r_t a_t + M_{1,t} - c_t] + r_t\frac{\partial V_t(a)}{\partial a} + \frac{\mathbb{E}_t[d(\partial V_t(a)/\partial a)]}{\partial t}
\end{aligned}$$

$$\Rightarrow (\rho - r_t)\frac{\partial V_t(a)}{\partial a} = \psi a_t^{-\gamma_b} + \frac{\partial^2 V_t(a)}{\partial a^2}\frac{da}{dt} + \frac{\mathbb{E}_t[d(\partial V_t(a)/\partial a)]}{\partial t}$$

The total time derivative of the expected shadow price of consumption $\frac{\partial V_t(a)}{\partial a}$ is

$$\frac{\mathbb{E}_t[d(\partial V_t(a)/\partial a)]}{dt} = \frac{\partial^2 V_t(a)}{\partial a^2}\frac{da}{dt} + \frac{\mathbb{E}_t[\partial(\partial V_t(a)/\partial a)]}{\partial t}$$

such that the shadow price evolves according to

$$\Rightarrow (\rho - r_t)\frac{\partial V_t(a)}{\partial a} = \psi a_t^{-\gamma_b} + \frac{\mathbb{E}_t[d(\partial V_t(a)/\partial a)]}{dt}$$

Plugging in the first-order condition,

$$\Rightarrow (\rho - r_t)c_{1,t}^{-\gamma} = \psi a_t^{-\gamma_b} + \frac{\mathbb{E}_t[d(c_{1,t}^{-\gamma})]}{dt}$$

where with the chain rule, $\frac{\mathbb{E}_t[d(c_{1,t}^{-\gamma})]}{dt} = -\gamma c_{1,t}^{-\gamma-1}\frac{\mathbb{E}_t[dc_{1,t}]}{dt}$. Rearranging,

$$\frac{\mathbb{E}_t[dc_{1,t}]}{dt}\frac{1}{c_{1,t}} = \gamma^{-1}\left[r_t + \psi c_{1,t}^{\gamma} a_t^{-\gamma_b} - \rho\right].$$

Appendix A.3.1. The saver household's linearized policy function

The saver household's budget constraint states that

$$\frac{da}{dt} = r_t a_t + y_t - c_t$$

where a is the household's asset position, and y_t is their total income (including transfers). Using $e^{-\int_\tau^t r_s ds}$ as an integrating factor,

$$e^{-\int_\tau^t r_s ds}\frac{da}{dt} - e^{-\int_\tau^t r_s ds} r_t a_t = e^{-\int_\tau^t r_s ds}[y_t - c_t]$$

$$\Rightarrow \quad \frac{d}{dt}\left[e^{-\int_\tau^t r_s ds} a_t\right] = e^{-\int_\tau^t r_s ds}[y_t - c_t]$$

Integrating forward to time T,

$$\int_\tau^T \frac{d}{dt}\left[e^{-\int_\tau^t r_s ds} a_t\right] dt = \int_\tau^T e^{-\int_\tau^t r_s ds}[y_t - c_t]dt$$

such that

$$e^{-\int_\tau^T r_s ds} a_T - e^{-\int_\tau^\tau r_s ds} a_\tau = \int_\tau^T e^{-\int_\tau^t r_s ds}[y_t - c_t]dt$$

Thus

$$a_\tau = \int_\tau^T e^{-\int_\tau^t r_s ds}[c_t - y_t]dt + e^{-\int_\tau^T r_s ds} a_T$$

And since the consumer's TVC and no-Ponzi condition stipulates $\lim_{T\to\infty}\mathbb{E}_\tau[e^{-\int_\tau^T r_s ds} a_T] = 0$,

$$a_t = \mathbb{E}_t\left[\int_t^\infty e^{-\int_t^\tau r_s ds}[c_\tau - y_\tau]d\tau\right]$$

where I have interchanged the τ and t indexes, for clarity. Households choose assets to fund the expected present value of their consumption that their expected future income will not cover.

$$\mathbb{E}_t\left[\int_t^{\infty} e^{-\int_t^{\tau} r_s ds} c_\tau d\tau\right] = a_t + \mathbb{E}_t\left[\int_t^{\infty} e^{-\int_t^{\tau} r_s ds} y_\tau d\tau\right]$$

Log linearizing around the NSS,

$$\mathbb{E}_t\left[\int_t^{\infty} e^{-(\tau-t)r-\int_t^{\tau}\widehat{r}_s ds} c e^{\widehat{c}_\tau} d\tau\right] = a e^{\widehat{a}_t} + \mathbb{E}_t\left[\int_t^{\infty} e^{-r(t-T)-\int_t^{\tau}\widehat{r}_s ds} y e^{\widehat{y}_\tau} d\tau\right]$$

$$\mathbb{E}_t\left[\int_t^{\infty} e^{-(\tau-t)r}\left(1-\int_t^{\tau}\widehat{r}_s ds\right) c(1+\widehat{c}_\tau) d\tau\right] = a e^{\widehat{a}_t} + \mathbb{E}_t\left[\int_t^{\infty} e^{-(\tau-t)r}\left(1-\int_t^{\tau}\widehat{r}_s ds\right) y(1+\widehat{y}_\tau) d\tau\right]$$

$$\mathbb{E}_t\left[c\int_t^{\infty} e^{-(\tau-t)r}\left(1-\int_t^{\tau}\widehat{r}_s ds+\widehat{c}_\tau\right) d\tau\right] = a(1+\widehat{a}_t) + \mathbb{E}_t\left[y\int_t^{\infty} e^{-(\tau-t)r}\left(1-\int_t^{\tau}\widehat{r}_s ds+\widehat{y}_\tau\right) d\tau\right]$$

$$\mathbb{E}_t\left[c\int_t^{\infty} e^{-(\tau-t)r}\left(\widehat{c}_\tau-\int_t^{\tau}\widehat{r}_s ds\right) d\tau\right] = a\widehat{a}_t + \mathbb{E}_t\left[y\int_t^{\infty} e^{-(\tau-t)r}\left(\widehat{y}_\tau-\int_t^{\tau}\widehat{r}_s ds\right) d\tau\right]$$

and since at steady state $y = c$,

$$\int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t\left[\widehat{c}_\tau\right] d\tau = \frac{a}{y}\widehat{a}_t + \int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t\left[\widehat{y}_\tau\right] d\tau$$

The Euler equation can then be log-linearized, with the understanding that $r = \rho$:

$$c\frac{\mathbb{E}_t[d\widehat{c}_t]}{dt} = \gamma^{-1}\left[r(1+\widehat{r}_t)-\rho\right] c(1+\widehat{c}_t) \;\Rightarrow\; \frac{\mathbb{E}_t[d\widehat{c}_t]}{dt} = \gamma^{-1}\widehat{r}_t$$

Taking expectations as of time $\tau < t$,

$$\frac{\mathbb{E}_\tau[d\widehat{c}_t]}{dt} = \gamma^{-1}\mathbb{E}_\tau[\widehat{r}_t]$$

such that integrating forward,

$$\underbrace{\int_\tau^{T}\frac{\mathbb{E}_\tau[d\widehat{c}_t]}{dt}}_{\mathbb{E}_\tau[\widehat{c}_T]-c_\tau} = \int_\tau^{T}\gamma^{-1}\mathbb{E}_\tau[\widehat{r}_t]dt$$

and returning to my standard time index notation,

$$\mathbb{E}_\tau[\widehat{c}_\tau] = c_t + \int_t^{\tau}\gamma^{-1}\mathbb{E}_t[\widehat{r}_s]ds$$

Substituting into the previous intertemporal budget constraint,

$$\int_t^{\infty} e^{-(\tau-t)r}\left(c_t + \int_t^{\tau} \gamma^{-1}\mathbb{E}_t[\widehat{r}_s]ds\right)d\tau = \frac{a}{y}\widehat{a}_t + \int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t\left[\widehat{y}_\tau\right]d\tau$$

$$\Rightarrow \quad \int_t^{\infty} e^{-(\tau-t)r}c_t d\tau + \int_t^{\infty} e^{-(\tau-t)r}\left(\int_t^{\tau} \gamma^{-1}\mathbb{E}_t[\widehat{r}_s]ds\right)d\tau = \frac{a}{y}\widehat{a}_t + \int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t\left[\widehat{y}_\tau\right]d\tau$$

Changing the order of integration in the second integral and solving the first:

$$\Rightarrow \quad \frac{1}{r}c_t + \gamma^{-1}\int_t^{\infty}\left(\int_s^{\infty} e^{-(\tau-t)r}d\tau\right)\mathbb{E}_t[\widehat{r}_s]ds = \frac{a}{y}\widehat{a}_t + \int_t^{\infty}\mathbb{E}_t\left[\widehat{y}_\tau\right]e^{-(\tau-t)r}d\tau$$

$$\Rightarrow \quad \frac{1}{r}c_t + \gamma^{-1}\frac{1}{r}\int_t^{\infty} e^{-(s-t)r}\mathbb{E}_t[\widehat{r}_s]ds = \frac{a}{y}\widehat{a}_t + \int_t^{\infty}\mathbb{E}_t\left[\widehat{y}_\tau\right]e^{-(\tau-t)r}d\tau$$

And since in the simple TANK model $r = \rho$ and $\widehat{r}_t = \widehat{i}_t - \widehat{\pi}_t$:

$$c_t = \rho\int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t\left[\widehat{y}_\tau\right]d\tau + \rho\frac{a}{y}\widehat{a}_t - \gamma^{-1}\int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t[\widehat{i}_\tau - \widehat{\pi}_\tau]d\tau \tag{A.4}$$

The forward-looking household's linearized MPC out of a NPV income shock of 1 is equal to ρ, if real interest rates are unchanged. This is also the household's MPC out of liquid wealth, where the liquid wealth change is also in terms of a percentage of steady state income. Note that from the perspective of when a shock is realized, $\widehat{a}_t = 0$ if the stock of the household's savings does not jump on impact.

Appendix A.4. Combining Equations

Suppose nominal interest rates are fixed and the path of surpluses is exogenously set for active fiscal policy. Then, the second term is equal to the (negative) present present value of future inflation, which is from the section on the government budget deficit equal to the present discounted value of expected deficits.

$$c_t = \rho\int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t\left[\widehat{y}_\tau\right]d\tau - \gamma^{-1}\frac{T}{B}\int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t[T_\tau]d\tau \tag{A.5}$$

Deficits induce inflation which entail a reduction in real rates, stimulating intertemporal substitution and consumption apart from the .

Appendix B. Determinacy of the HANK Model Under Different Policy Environments

Both Auclert et al. (2023a) and Hagedorn (2024) propose tests for the uniqueness and determinacy of a linearized rational expectations model using a criterion based on Onatski (2006), which can handle models with theoretically infinite lags and leads. For a model with endogenous states y_t and exogenous states x_t that takes the form

$$\sum_{k=-\infty}^{\infty} A_k \mathbb{E}_t y_{t-k} = \Gamma x_t$$

Onatski (2006) proposes constructing the complex-valued criterion function

$$\det \widehat{A}(\lambda) = \det \left[\sum_{k=-\infty}^{\infty} A_k e^{ik\lambda} \right] \tag{B.1}$$

where $i = \sqrt{-i}$ is Euler's constant and k is the number of lags (such that the coefficients are presented going back in time relative to t). As such, $\widehat{A}(\lambda)$ is essentially the discrete[4] Fourier transform of the model's time indexed matrix coefficients, and so describes the phase and amplitude of different frequencies $\lambda \in [0, 2\pi]$ that generate the coefficients. He then defines the *winding number* of the criterion function as the contour integral of the function evaluated over $[0, 2\pi]$ – tantamount to evaluating the Z (Laplace) transformation of the coefficients over the unit circle in the complex plane – which quantifies how many times the graph of the function encircles the origin. For a large class of economic models the author terms "generic,"[5] the model has a unique solution if the

[4]For a continuous time system, the analogous model would be

$$\int_{-\infty}^{\infty} A_\tau \mathbb{E}_t x_{t-\tau} d\tau = \Gamma z_t$$

and a criterion that uses the continuous Fourier transform

$$\widehat{A}(\lambda) = \int_{-\infty}^{\infty} A_\tau e^{i\lambda\tau} d\tau.$$

However because my numerical solution of the model is discretized for time grid points of a fixed interval, this is essentially tantamount to using the discrete formulation, but with the rotation re-scaled by the size of the time step Δt, as $t = \Delta t \times k$, such that the criterion function becomes

$$\det \widehat{A}(\lambda) = \det \left[\sum_{k=-\infty}^{\infty} A_k e^{ik\lambda\Delta t} \Delta t \right].$$

Since the sequence space numerical solution of my model is essentially a discrete-time system on the computer, I evaluate its Onatski (2006) criterion as one would a discrete-time model.

[5]Onatski (2006) defines models as "generic" where all of the time shift components of the Weiner-Hopf factorization of the criterion, called partial indexes, are either zero or of the same sign.

winding number is zero such that the graph of the criterion function from $[0, 2\pi]$ does not enclose the origin.

As Auclert (2018) discusses, the intuition is similar to that of the Blanchard and Kahn (1980) conditions. If the winding number is equal to zero, then the criterion function has as many zeros as poles outside of the unit circle via the Cauchy argument principle, and therefore essentially has as many explosive roots as non-predetermined variables. If the function wraps around the origin counter-clockwise (such that it has a positive winding number) then the model has no solution; if it wraps around the origin counter-clockwise (such that it has a negative winding number), then there exist a multiplicity of solutions.

However, the original Onatski (2006) criterion was designed for time-invariant systems, where only the difference in time determined the system's interaction with its own leads and lags. For the sequence-space Jacobian method proposed by Auclert et al. (2021), this requires that the sequence space Jacobian matrix is Toeplitz, a property that it does not generally have. However, Auclert et al. (2023a) note that HANK models typically have “quasi”-Toeplitz structure, in that the response of the system at time t to a future perfect foresight shock at time s becomes largely invariant to the precise date s and instead only depends on $s - t$. Different future shocks, in other words, begin to look like time-transposed versions of one another. Auclert et al. (2023a) then argue that they can approximate

$$A_k = \lim_{t \to \infty} A_{k,t}$$

where A_k are the elements of the sequence-space Jacobian matrix that the endogenous states at time t to their values k periods in the past. The authors then impose the Onatski (2006) criterion on the system's response to a future shock and argue that it provides a check for the determinacy of the system overall.

Hagedorn (2023) takes a similar, but slightly different, approach. The author employs a dimension reduction routine to the equilibrium and models the economy such that agents do not track the whole distribution, but instead only track the aggregate level of assets. In doing so, the agents forecast prices in the economy under the assumption that the future distribution looks like the steady state one – but with all of the other agents' wealth scaled up or down by the aggregate asset position. If the aggregate asset position is included as a state variable, the simplified system becomes truly Toeplitz – such that the Onatski (2006) criterion may be straightforwardly applied.

Lastly, Bayer and Luetticke (2020) uses a completely different numerical approach and suggests

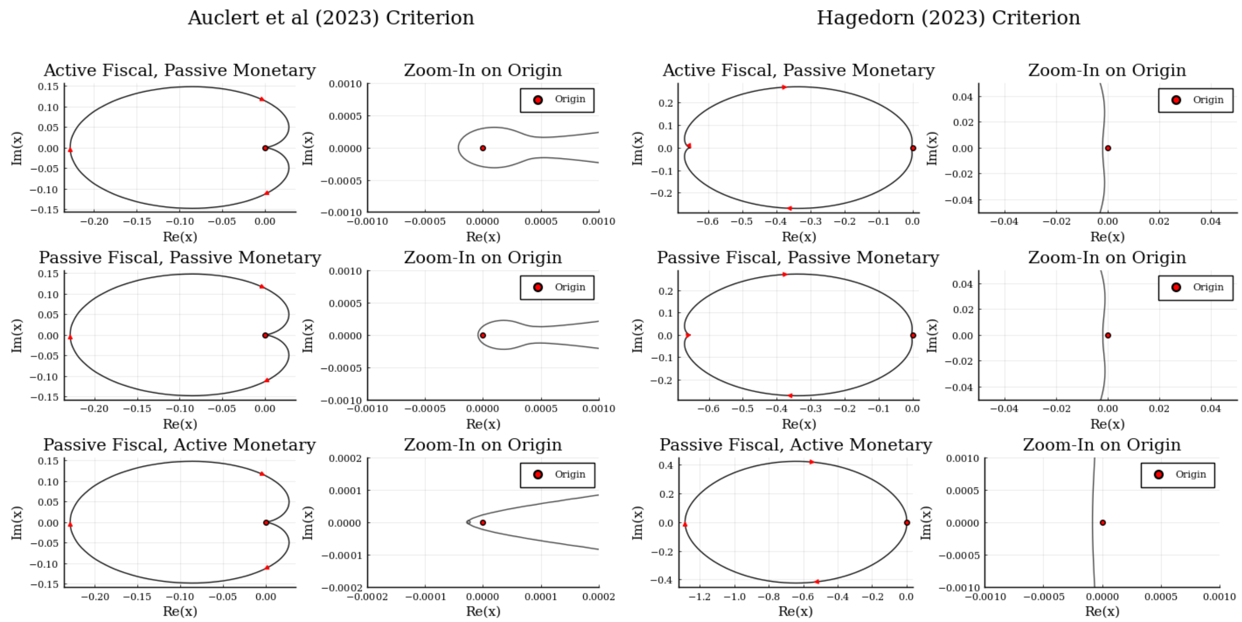


Figure B.6: Onatski criterion for a sequence-space solution of a HANK model, both with the Auclert et al. (2023a) determinacy criterion and the Hagedorn (2023) criterion, for different active/passive fiscal and monetary settings. Both confirm that the HANK model has a unique sequence solution. None of the criterion wind around the origin, implying that the model has a unique solution under the different calibrations listed in Table 3. Arrows denote the direction of the graph around the origin.

solving HANK models in state-space using a dimension reduction strategy similar to the one employed by Reiter (2009). In the last section of this appendix, I detail the steps and how it may be used to solve my HANK model. They argue that the dimension-reduced model's stability and determinacy may then be evaluated as in Blanchard and Kahn (1980): a system has a unique solution if it has as many explosive (positive) eigenvalues as it has jump variables. I solve my state space model using the QZ decomposition suggested by Sims (2002) and consider its generalized eigenvalues.

For each of my three active fiscal/passive monetary, passive fiscal/passive monetary, and passive monetary/active fiscal settings, I check the determinacy of my model in all three ways. The Bayer and Luetticke (2020) results are straightforward; my dimension-reduced system has as many explosive eigenvalues as it has forward-looking control variables. The graphs of the criterion functions for both the Auclert et al. (2023a) and Hagedorn (2023) methodologies are displayed in Figure B.6. None of the graphs encircle the origin.

All three different methodologies suggest that in each of my three HANK calibrations, the model exhibits determinacy.

Appendix B.1. Brief Summary of Intuition Provided by Auclert et al. (2023a)

To briefly sketch the intuition of Onatski (2006)'s methodology, Auclert et al. (2023a) note that Onatski (2006) essentially recommends taking determinant of the z-transformation (discrete-time Laplace transformation) of the sequence of the model's coefficients, a common technique used in signal processing:

$$\det \widehat{A}(z) = \det \left[\sum_{k=-\infty}^{\infty} A_k z^k \right]$$

where $z = e^{\alpha + i\omega} \in \mathbb{C}$ is a point in the complex plane that describes both a sinusoidal frequency and exponential magnitude. As noted in Auclert et al. (2023a), the contour integral of the graph of $\det \widehat{A}(z)$ evaluated over the unit circle $|z| = 1$ is known as the function's *winding number*, as it counts the number of times the function wraps around the origin counter-clockwise. They further denote the number of zeros of $\det \widehat{A}(z)$ inside the unit circle as N; these are essentially stable roots. r predetermined variables affect the current state in the z-transformation via a time shift of z^{-r}; with the fundamental theory of algebra, z^r has r roots, such that the criterion function then has r stable poles. They then note that via Cauchy's argument principle,

$$\text{wind} \det \widehat{A}(z) = \frac{1}{2\pi i} \oint_{\det \widehat{A}(C)} \frac{dz}{z} = N - r$$

If $\det \widehat{A}(z)$ does not wrap around zero, then $Z - P = 0$ and the number of zeros in the unit circle is equal to the number of poles, and the system admits a unique solution. As a corollary, the number of stable roots is equal to the number of predetermined state variables, matching the Blanchard and Kahn (1980) conditions for existence and determinacy.

Appendix B.2. Onatski (2006) and Partial Indexes

Onatski (2006) constructs his criterion using the Weiner-Hopf factorization of $\widehat{A}(\lambda)$ into three components: an explosive root component $\widehat{A}_+(\lambda)$, a stable component $\widehat{A}_-(\lambda)$, and a component that only pertains to the time shift of the coefficients (which can be accomplished by multiplication or division of the z-transform by a factor of $e^{\lambda i}$) $A_0(\lambda)$. All together,

$$\widehat{A}(\lambda) = \widehat{A}_-(\lambda)\widehat{A}_0(\lambda)\widehat{A}_+(\lambda)$$

He notes that the time shift component $\widehat{A}_0(\lambda)$ is a diagonal matrix $\text{diag}(e^{i\lambda k_1}, \ldots, {}^{i\lambda k_n})$, where n is the number of variables in x_t and $(k_1, \ldots, k_n)$ are the number of periods each variable is lagged time shift component of the factorization, known as the "partial indexes." Onatski (2006) calls a model "generic" if its winding numbers are all of the same sign or zero. Then, a winding number of zero implies that all of the partial indexes of the model are zero as well.

From his paper, Proposition 1 then states that if the partial indexes are all i) equal to zero, then the model solution exists and is unique, ii) weakly negative, with at least one strictly negative, then the model is indeterminate, and iii) weakly positive, with at least one strictly positive, then a solution does not exist. He then shows that, because the winding number of the root components is always zero and the time shift matrix containing the partial indexes is diagonal,

$$\begin{aligned}
\operatorname{wind}\det \widehat{A}(\lambda) =& \operatorname{wind}(\det[\widehat{A}_-(\lambda)]\det[\widehat{A}_0(\lambda)]\det[\widehat{A}_+(\lambda)] \\
=& \operatorname{wind}\det[\widehat{A}_-(\lambda)] + \operatorname{wind}\det[\widehat{A}_0(\lambda) + \operatorname{wind}\det[\widehat{A}_+(\lambda)] \\
=& \operatorname{wind}\det \widehat{A}_0(\lambda) \\
=& \operatorname{wind}\exp\left(\lambda i \sum_{j=1}^{T} k_j\right) = \sum_{j=1}^{T} k_j
\end{aligned}$$

The winding number is equal to the sum of partial indexes. Thus, if a model is generic, then the winding number will only be equal to zero if the partial indexes are all zero, negative if the partial indexes are all weakly negative, and positive if the partial indexes are all weakly positive.

Appendix C. TANK Experiments

As noted in the main text of the paper, the simple TANK model in Section 3 is determined due to the FTPL, while Hagedorn (2023) notes that HANK models with passive monetary policy are determinate via the demand theory of the price level described in Hagedorn (2016). In this section of the appendix, I modify a TANK model to exhibit a version of the DTPL as well, however, with little change to its qualitative dynamics. The models are all solved in state-space form with a Schur decomposition, as in Sims (2002).

A two-agent TANK model with a saver household and a spender household is the simplest framework to explore household heterogeneity and its implications for output and inflation. Auclert et al. (2018) and Kaplan and Violante (2018) also note that a TANK model with bonds in the utility function (abbreviated to TANK-BIU) also generates a profile of intertemporal MPCs (iMPCs) that is highly similar to that of a HANK model, as the bond utility term mimics the more complicated precautionary savings motives present in incomplete market models.

Relatedly, Auclert (2018) notes that a bonds-in-the-utility model violates Ricardian equivalence[6] and presents an endogenous relationship between the real interest rate and the path of government debt, allowing the model to display a version of the DTPL. A TANK-BIU framework thus captures the essential elements of both a HANK model's MPC heterogeneity and price level determination. It is the starting point of my analysis before I show that its implications are robust to the calibrated full-HANK setting.

In the TANK model, time $t \geq 0$ is continuous. At a high level, there are two types of representative households: an inter-temporal consumption smoother (a "saver") and an agent that spends all of its contemporaneous income (a "spender"). The rest of the model is identical to the HANK presented in the rest of the paper. The numerical solutions are all for a perfect foresight environment. Once the shock is realized, the transition dynamics are deterministic and known to the agents in the model.

Appendix C.1. Households

Like in the simple Section 3 TANK model, a $1-\mu$ fraction of households behave as savers (labeled "1") and solve an intertemporal optimization problem where they maximize their intertemporal

[6] In either case, when agents have more assets and the government has issued more liabilities, households want to consume more, because either they mechanically want to substitute from bonds to consumption in the TANK, or because they feel better insured and want to substitute to more consumption in the HANK.

utility subject to a flow budget constraint and a no-Ponzi condition:

$$
\begin{aligned}
&\max_{(c_{1,t})_{t\geq 0}} \mathbb{E} \int_0^\infty e^{-\rho t} \left[\frac{c_{1,t}^{1-\gamma}}{1-\gamma} - \frac{h_{1,t}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \psi \frac{a_t^{1-\gamma_b}}{1-\gamma_b} \right] dt \\
&\text{s.t. } \frac{da_t}{dt} = (1-\tau) w_t h_{1,t} + r_t a_t + M_{1,t} - c_t \\
&\lim_{T\to\infty} \mathbb{E}[e^{-\int_0^T r_t dt} a_T] \geq 0
\end{aligned}
\tag{C.1}
$$

The inverse of γ_b sets the agent's marginal elasticity of utility with respect to holding real liquid assets (which in equilibrium are on net government bonds). If $\psi > 0$, then it is possible for the Hagedorn (2016) DTPL to provide determinacy in the TANK framework, as it does in the HANK world. If I set $\psi = 0$, the DTPL is no longer relevant and determinacy can be ensured by alternative mechanisms, like the FTPL when fiscal policy is active.

Savers in the TANK model thus follow an Euler equation, derived in Appendix A.3:

$$
\frac{\mathbb{E}_t[dc_{1,t}]}{dt} \frac{1}{c_{1,t}} = \gamma^{-1} \left(r_t + \psi \frac{a_t^{-\gamma_b}}{c_{1,t}^{-\gamma}} - \rho \right).
$$

The Euler equation is standard, except for the fact that bonds can enter into the agents' utility function to provide a liquidity-like effect. Since they are consumption smoothing and forward-looking, growth in the consumption of wealthy saver agents can be characterized by changes in their real asset position and the equilibrium real interest rate.

The remaining μ measure of households are hand-to-mouth spenders (labeled m) who are exogenously constrained to consume all of their income as soon as it is received. This income is composed of their real wage w_t times hours worked $h_{m,t}$ less a constant income tax rate τ, plus net transfers $M_{m,t}$, such that

$$
c_{m,t} = (1-\tau) w_t h_{m,t} + M_{m,t}. \tag{C.2}
$$

While these households are constrained, I assume that their preferences for labor and consumption are the same as those of the savers.

Appendix C.2. Firms and Price Setting

The Phillips curve with only two agents becomes

$$
\frac{\mathbb{E}_t[d\pi_t]}{dt} = \rho \pi_t - \frac{\varepsilon_L}{\theta_w} L_t \left\{ (1-\mu) \left[h_t^{\frac{1}{\eta}} - \frac{\varepsilon_L}{\varepsilon_L - 1} w_t c_t^{-\gamma} \right] + \mu \left[(h_t^m)^{\frac{1}{\eta}} - \frac{\varepsilon_L}{\varepsilon_L - 1} w_t (c_t^m)^{-\gamma} \right] \right\}. \tag{C.3}
$$

Inflation today is related to both expected inflation and the average cross-sectional wedge between the disutility of labor and the utility of working for wages, marked up because the unions internalize the effect of supplying more labor on their wage rate.

Appendix C.2.1. Taxes

As in the main HANK model, the primary budget surplus is

$$T_t = \tau w_t L_t - M_t. \tag{C.4}$$

In the NSS, the government again balances the budget and rebates excess tax revenue uniformly back to both households:

$$M_{NSS} = \tau w L_{NSS} - r_{NSS} B_{NSS}.$$

Outside of the steady state, transfers to households of type i can be written as the sum of a shock to transfers to all agents, plus a shock in transfers to only agents of type i:

$$M_{i,t} = M_{NSS} + 4Y_{NSS} \times \left(\zeta_{\text{All},t} + \frac{1}{\mu_i}\zeta_{i,t} \right) - \kappa_B \left(\frac{q_{NSS}}{q_t} B_t - B_{NSS} \right). \tag{C.5}$$

μ_i is the share of agents of type i while $\zeta_{i,t}$ is the shock to transfers of type i.

Aggregate transfers are then

$$M_t = (1-\mu)M_{1,t} + \mu M_{m,t}. \tag{C.6}$$

Appendix C.3. Market Clearing

Aggregate consumption $C_t = (1-\mu)c_{1,t} + \mu c_{m,t}$ is equal to aggregate output:

$$Y_t = C_t \tag{C.7}$$

and total hours worked are uniform across households:

$$h_{m,t} = h_{1,t} = L_t. \tag{C.8}$$

The asset market clears when net private wealth equal to aggregate government debt:

$$(1-\mu)a_t = B_t. \tag{C.9}$$

All other equations are exactly as they appear in the full HANK model.

Appendix C.4. TANK Equilibrium

An equilibrium given a sequence of aggregate shocks $(\zeta_t)_{t\geq 0}$, a fixed distribution of agents given by $(\mu, 1-\mu)$, and an initial debt level B_0 is therefore a collection of sequences of macroeconomic aggregates

$$(C_t, L_t, Y_t, B_t)_{t\geq 0}$$

and household-level variables and prices

$$(c_{m,t}, c_{s,t}, h_{m,t}, h_{s,t}, M_{m,t}, M_{s,t}, w_t, r_t, i_t, \pi_t, q_t)_{t\geq 0}$$

where

i. saver consumption choices $(c_{s,t})_{t\geq 0}$ solve (C.1) given prices and aggregates, while spender households' $(c_{m,t})_{t\geq 0}$ are consistent with (C.2)

ii. labor allocations $(h_{t,m}, h_{t,s})$ are consistent with the union rule (C.8)

iii. inflation π_t is consistent with the unions' maximization problem and resulting wage Phillips Curve (C.3)

iv. nominal government bond prices $(q_t)_{t\geq 0}$ are consistent with (12)

such that

1. Macro aggregates $(Y_t, C_t)_{t\geq 0}$ are consistent with production (9) and aggregation $C_t = \mu c_{m,t} + (1-\mu)c_{s,t}$

2. real wages w_t are constant and real rates r_t obey the Fisher equation $r_t = i_t - \pi_t$

3. nominal interest rates $(i_t)_{t\geq 0}$ follow the central bank's policy rule (16)

4. Government taxes and transfers across the population and over time $(M_{m,t}, M_{s,t})_{t\geq 0}$ follow the rule (C.5) and aggregate to M_t and T_t via (C.6) and (13)

5. Government debt B_t given taxes T_t and real rates r_t evolves according to (11)

6. The asset market clears, as in (C.9). By Walras' law, this also implies goods market clearing (C.7).

Appendix C.5. TANK Model Calibration

The TANK model is calibrated exactly the same as in the HANK model, except for the choice of ρ and TANK model-specific parameters like ψ and γ_b. These are chosen depending on the active/passive fiscal and monetary policy mix, as displayed in Table C.5.

The columns are grouped by whether the model does not have bonds in the utility function (under the heading "TANK"), or whether the model does have bonds in the utility function ("TANK-BIU"). From there, the models are separated based on their policy regime type: "PF/AM" stands for Passive Fiscal/Active Monetary (the standard New Keynesian regime), "AF/PM" stands for Active Fiscal/Passive Monetary (the FTPL in the model without bonds in the utility function, and the DTPL in the model with bonds in the utility function). "PF/PM" stands for a setting in which both fiscal and monetary policy are passive; the DTPL can still provide a determinate equilibrium in this case, even if the FTPL cannot.

Preferences are calibrated to be consistent with $r = 0.005$ in the non-stochastic steady state, such that nominal and real interest rates are targeted to 2% annually. This means setting $\rho = 0.005$ for the models where bonds do not appear in the utility function (the first two columns). For the last three columns, where bonds do appear in the utility function, the model is more closely analogous to an incomplete market HANK model. As such, I set $\rho = 0.023$ to be consistent with my HANK model and then set $\gamma_b = 2.5$, a value which Kaplan and Violante (2018) note leads TANK models to have similar MPCs to HANK ones. I then adjust ψ to achieve a steady state annual interest rate of 2%. I also assume $\mu = 0.26$ across all of the specifications, such that 26% of households are spenders and 74% are savers, to be consistent with the number of borrowing-constrained households in the full HANK model's non-stochastic steady state.

For the different policy mixes, I make the policies active fiscal by setting $\kappa = 0$ and passive fiscal by setting $\kappa = 0.01$. I similarly make monetary policy active by setting $\phi_\pi = 1.05$ and passive by setting $\phi_\pi = 0$.

In each of the TANK models and policy regimes, I solve the models using standard state-space techniques, with a Schur decomposition as in Sims (2002). I determine each equilibrium to exist and be unique by establishing that the number of jump variables is equal to the number of explosive generalized eigenvalues.

Table C.5: Model Parameters (by Policy Regime and Model Type)

		TANK		TANK-BIU		
			FTPL		DTPL	
	Symbol	PF/AM	AF/PM	PF/AM	AF/PM	PF/PM
Panel A: TANK Parameters						
Quarterly Time Discounting	ρ	0.005	0.005	0.021	0.021	0.021
Share of Spender Households	μ	0.26	0.26	0.27	0.27	0.27
Bond Utility Weight	ψ	0	0	0.25	0.25	0.25
Bond Utility Elasticity	γ_b	-	-	2.5	2.5	2.5
Panel B: Policy Mix						
Auto Fiscal Adj.	κ	0.01	0	0.01	0	0.01
Taylor Rule Coef.	ϕ_π	1.10	0	1.10	0	0

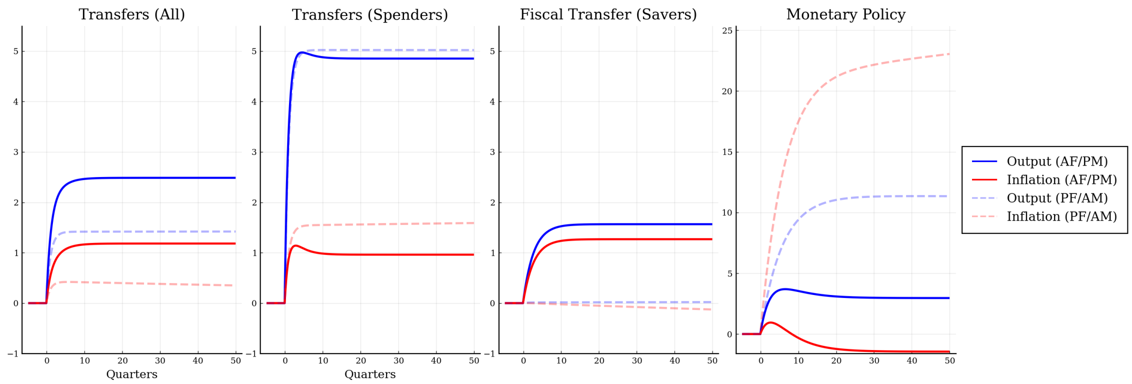
Appendix C.6. TANK Results

The results of the simpler TANK models are qualitatively silimar to their HANK model counterparts, for both the monetary and fiscal policy experiments. Figure C.7 displays the responses of $\mathcal{C}Y$ and $\mathcal{C}\pi_t$ following 1% increases in transfers to different segments of the population. Panel A refers to the TANK model without bonds in the utility function ($\psi = 0$), while Panel B includes bonds in the utility function ($\psi = 0.25$). Blue lines refer to the change in real GDP, while red lines refer to the change in inflation. Solid lines refer to the active fiscal/passive monetary mix, while dashed lines refer to the more conventional passive fiscal/active monetary one. Dotted lines refer to the setting where both types of policy are passive; these series are displayed only for the TANK-BIU model since the model is still determinate via the DTPL but not via the FTPL.

For both the TANK and TANK-BIU models, output and inflation dynamics are very similar so long as fiscal policy is active. The accumulated output gap reaches about 2% following a 1% of GDP disbursal of deficit-financed transfers to all households, nearly 5% following a disbursal to zero-wealth spender households, and about 1.5% following a disbursal to wealthy savers. Inflation, in all of the fiscal transfer scenarios, accumulates just to a little over 1.1%. It barely matters if fiscal policy is also made slightly passive in the DTPL environment if the speed of the fiscal adjustment is slow and monetary policy remains passive; the dotted lines are essentially on top of the solid ones in Panel B, to the point that they require close inspection to see.

If monetary policy is active and fiscal policy is passive, however, the total magnitude of the deficit-financed transfers is no longer sufficient to qualitatively characterize the amount of inflation. Transfers to wealthy savers barely move output and inflation in the baseline TANK (panel A), as the savers are Ricardian and know i) real rates will rise in response to inflation and not fall

Panel A: TANK



Panel B: TANK-BIU

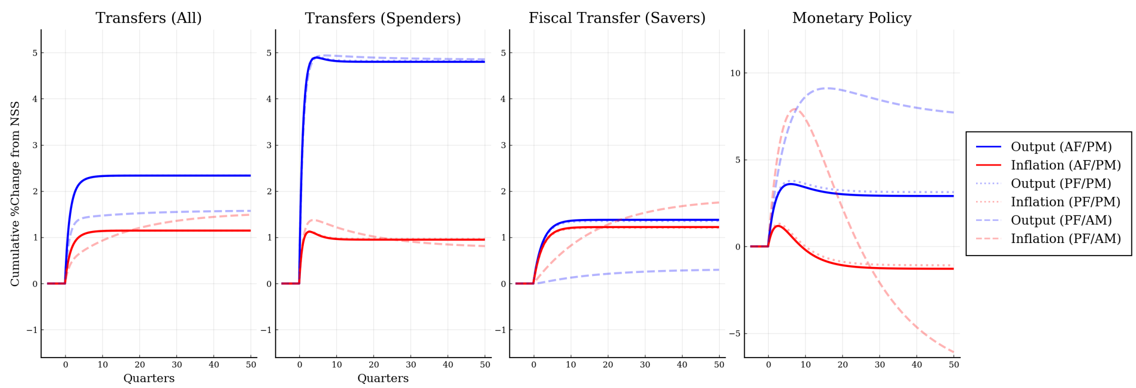


Figure C.7: Cumulative impulse response functions in a TANK model, with and without bonds in the utility function (BIU). Shocks include 1% increases in transfers to all agents, spender agents, and saver agents, respectively. The last plot in each panel depicts the response to a 1% reduction in the central bank's nominal interest rate target.

to erode away government debt, and ii) debt today necessitates fiscal readjustments and taxes in the future.[7] Conversely, transfers to poorer agents imply both more output and more inflation relative to transfers to the savers, as opposed to similar amounts of inflation and more output when monetary policy is passive.

In both the DTPL and the FTPL, nominal government debt serves as a nominal anchor for inflation when monetary policy is passive. In FTPL, if the government unexpectedly signals that it will begin running deficits, then households will not want to hold government bonds unless either i) the price level rises immediately or ii) inflation drives down the real interest rate over time to make the present value of future surpluses equal to the current bonds outstanding. If the price level follows any other path, debt grows explosively and violates the savers' transversality constraint.

[7]The slight changes in output and prices that do occur stem from the fact that the wealthy households know that the poor spender households pay part of the future budget-balancing taxes.

Similarly, for the DTPL, more real bonds generate a liquidity or wealth effect for households in partial equilibrium, leading them to want to save less and spend more. Not everyone can be a net bond spender in general equilibrium, however, as every seller necessitates a buyer; the price level has to rise to devalue the bonds and clear the asset market, either immediately or over time. If it did not, then to motivate households to hold higher real balances in the current period, the return to doing so would have to rise, which would again lead to a divergent path of debt, ruled out as an equilibrium by the transversality condition.[8] If inflation initially overshoots the amount of inflation required to control the ratio of debt to GDP, as it does in the preceding fiscal simulations displayed in C.7, then households react to the erosion of their assets by trying to re-accumulate them through saving, instigating a gradual recession and deflation in equilibrium.

The mechanisms have strong implications for fiscal policy. Nominal debt determines the path of prices. MPC heterogeneity changes the way nominal debt can change real employment and output. Different redistribution policies can thus lead to different ratios of inflation to output.

[8]Although note that a transversality condition and infinitely-lived households are not required for the DTPL; see Hagedorn (2024), where the DTPL pins down the price level in an overlapping generations setting.

Appendix D. HANK Model Derivations

Appendix D.1. Bond Math

Appendix D.1.1. General Maturities and Formula Derivations

To elaborate more upon the structure of government debt in my model, I more generally assume that the government is able to borrow using long-term nominal bonds of any maturity τ, as in Cochrane (2018). As such, it can pay off existing nominal debt $\tilde{B}$ maturing at time t by either running a primary surplus or by selling new bonds with a maturity of τ at a price of $Q^B_{t,t+\tau}$. The debt flow equation is thus

$$\underbrace{\tilde{B}_{t,t}dt}_{\substack{\text{Debt}\\\text{maturing}\\\text{at time } t}} = \underbrace{p_t(T_t - G_t)dt}_{\text{Surplus}} + \underbrace{\int_0^\infty Q^B_{t,t+\tau} d\tilde{B}_{t,t+\tau} d\tau}_{\substack{\text{Financing from}\\\text{new bond sales}}}$$

I denote the real value of total government debt outstanding at time t as B_t, such that

$$B_t \equiv \frac{\int_0^\infty Q^B_{t,t+\tau}\tilde{B}_{t,t+\tau}d\tau}{p_t}$$

I next assume that bonds are purchased and priced not directly by households, but rather by a risk-neutral profit-maximizing investment fund that buys debt from the government and sells shares to the public. The central fiscal theory equation alluded to in the introduction of this paper therefore takes the form presented in Cochrane (2018):

$$\underbrace{\frac{\int_0^\infty Q^B_{t,t+\tau}\tilde{B}_{t,t+\tau}d\tau}{p_t}}_{\substack{\text{Real debt}\\\text{outstanding}}} = \mathbb{E}_t\left[\int_t^\infty e^{-\int_t^\tau r_s ds}[T_\tau - G_\tau]d\tau\right]$$

Each household that holds liquid assets by holding shares in the fund thus effectively owns a cross-sectional slice of the entire government portfolio, and receives whatever interest payments are distributed and absorbs whatever capital gains and losses the government debt accrues.

For the bond portfolio, the total *real* return is the real capital gain on each bond type, weighted by the value of the bonds held, divided by the real value of the entire portfolio:

$$dR_t = \frac{\int_0^\infty \left[d\left(\frac{Q_{t,t+\tau}}{p_t}\right)/\frac{Q_{t+\tau}}{p_t}\right]\frac{Q_{t,t+\tau}}{p_t}\tilde{B}_{t,t+\tau}d\tau}{B_t}$$

$$\Rightarrow \quad B_t dR_t = \int_0^\infty d\left(\frac{Q_{t,t+\tau}}{p_t}\right)\tilde{B}_{t,t+\tau} d\tau$$

Such that

$$dB_t = d\left[\frac{\int_0^\infty Q_{t,t+\tau}\tilde{B}_{t,t+\tau} d\tau}{p_t}\right] = \underbrace{\frac{\int_0^\infty Q_{t,t+\tau} d\tilde{B}_{t,t+\tau} d\tau}{p_t} - \frac{\tilde{B}_t}{p_t} dt}_{-(T_t - G_t)dt} + \underbrace{\int_0^\infty \tilde{B}_{t,t+\tau} d\left(\frac{Q_{t,t+\tau}}{p_t}\right) d\tau}_{B_t dR_t}$$

It thus follows that

$$dB_t = -(T_t - G_t)dt + B_t dR_t$$

The first term is the primary deficit, while the second is the ex-post real rate of return on the bond portfolio. This ex-ante return will then be the expected return on the nominally riskless bonds, plus whatever capital gain has been unexpectedly accrued over the time increment.

Again as in Cochrane (2018), I make the simplifying assumption that the government issues and rolls over debt such that the density of of government liabilities by maturity is always exponentially distributed with a rate of ω, such that the cumulative distribution of outstanding government treasury maturities τ is $CDF(\tau) = 1 - e^{-\omega\tau}$ and the density function is $PDF(\tau) = \omega e^{-\omega\tau}$. Additionally, I make the simplifying assumption that in the non-stochastic steady state of the model, all households effectively hold the same representative slice of government debt by owning shares of a competitive profit-maximizing mutual fund, just in varying amounts. For an individual holding a unitary share of the total government portfolio, their assets entitle them to a payment of ωdt almost immediately (this is the shortest-term debt being repaid), plus payments of $\omega e^{-\omega\tau} dt$ for all periods thereafter. The entire bond portfolio is then effectively a perpetuity which pays out a geometrically declining coupon $\omega e^{-\omega\tau} dt$ at each time $t + \tau$ for the rest of time.

The nominal bond price of the entire portfolio will then be

$$q_t = \int_0^\infty e^{-\tau y_t} \omega e^{-\omega\tau} d\tau = \int_0^\infty \omega e^{-\tau(\omega + y_t)} d\tau = -\frac{\omega}{\omega + y_t} e^{-u}|_0^\infty = \frac{\omega}{\omega + y_t}$$

The nominal rate of return on the bond will be the the dividend yield, plus the capital gain.

$$dR_t^{nom} = \frac{(\omega - \omega q_t)dt + dq_t}{q_t} = y_t dt + \frac{dq_t}{q_t}$$

It then follows that if the ex-ante nominal rate of return is dR_t^{nom} is $i_t dt$ in expectation

$$i_t dt = \mathbb{E}_t[dR_t^{nom}] = y_t dt + \frac{\mathbb{E}_t[dq_t^B]}{q_t^B}$$

I define $\delta_{qB,t} = dq_t - \mathbb{E}_t[dq_t]$ as the unexpected gain in bond prices, which must in turn be equal to the ex-post nominal rate of return minus the expected (ex-ante) one:

$$\frac{\delta_{qB,t}}{q_t} \equiv dR_t^{nom} - i_t dt = \frac{dq_t^B - \mathbb{E}_t[dq_t^B]}{q_t^B}$$

Since the nominal rate will be the real one, plus inflation:

$$dR_t^{nom} = dR_t + \pi_t dt$$

$$\Rightarrow \frac{\delta_{qB,t}}{q_t} - \pi_t dt = dR_t - i_t dt$$

$$\Rightarrow \quad dR_t = \frac{\delta_{qB,t}}{q_t} + (i_t - \pi_t) dt$$

The valuation equation becomes

$$dB_t = -(T_t - G_t)dt + B_t \left[i_t - \pi_t \right] dt + \frac{\delta_{qB,t}}{q_t} B_t \tag{D.1}$$

To derive the equation governing nominal bond prices, it also follows that if

$$dR_t^n = \frac{\omega dt + dq_t^B}{q_t^B} - \omega dt$$

such that

$$q_t dR_t^n = \omega dt + dq_t^B - \omega q_t dt$$

then in expectation

$$E_t[dq_t] = q_t \left(\mathbb{E}_t[dR_t^n] + \omega dt - \frac{dt}{q_t} \right)$$

$$\Rightarrow \quad E_t[dq_t] = q_t \left(i_t + \omega - \frac{\omega}{q_t} \right) dt \tag{D.2}$$

and so bond prices evolve according to

$$dq_t = q_t \left(i_t + \omega - \frac{\omega}{q_t} \right) dt + \delta_{qB,t}$$

Appendix D.2. Wage Phillips Curve

This is a continuous-time version of Auclert et al. (2018), *The Intertemporal Keynesian Cross*. Say a labor-aggregator hires labor from households to create an aggregate unit of input labor:

$$L_{k,t} = \int_0^1 (z_i h_{ikt}) di$$

And labor from each union is differentiated with elasticity of substitution ε_ℓ:

$$L_t = \left(\int_0^1 L_{k,t}^{\frac{\varepsilon_\ell - 1}{\varepsilon_\ell}} dk \right)^{\frac{\varepsilon_\ell}{\varepsilon_\ell - 1}}$$

Let W_t be the nominal wage paid by employers to labor-aggregators, and let the labor-aggregator pay its workers a nominal wage of $W_{k,t}$. Labor-aggregating firms thus hire according to

$$\max_{\{L_{k,t}\}_{k \in [0,1]}} W_t \left(\int_0^1 L_{k,t}^{\frac{\varepsilon_\ell - 1}{\varepsilon_\ell}} dk \right)^{\frac{\varepsilon_\ell}{\varepsilon_\ell - 1}} - \int_0^1 W_{k,t} L_{k,t} dk$$

such that from the FOCs, the demand for labor from union k is

$$W_t \left(\int_0^1 L_{k,t}^{\frac{\varepsilon_\ell - 1}{\varepsilon_\ell}} dk \right)^{\frac{\varepsilon_\ell}{\varepsilon_\ell - 1} - 1} L_{k,t}^{-\frac{1}{\varepsilon_\ell}} - W_{k,t} = 0$$

$$W_t L_t^{\frac{1}{\varepsilon_\ell}} L_{k,t}^{-\frac{1}{\varepsilon_\ell}} = W_{k,t}$$

$$W_t L_t^{\frac{1}{\varepsilon_\ell}} = W_{k,t} L_{k,t}^{\frac{1}{\varepsilon_\ell}}$$

$$\Rightarrow \quad \frac{L_{k,t}}{L_t} = \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell}$$

Unions face nominal wage adjustment costs:

$$\frac{\theta_w}{2} \int_0^1 \pi_{w,k}^2 dk, \quad \text{where} \quad \pi_{w,k} = \frac{dW_{k,t}}{dt} \frac{1}{W_{k,t}}$$

The labor union k sets wages to maximize its members' lifetime utilities:

$$J_t^w(W_{k,t}) = \max_{\pi_{k,t}^w} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\int \int \left\{ \frac{c(a,z)^{1-\gamma}}{1-\gamma} - \frac{h(a,z)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\} \mu_t(a,z) da \ dz - \frac{\theta_w}{2} (\pi_{k,t}^w)^2 \right] dt$$

$$\text{s.t. } \frac{dW_t}{dt} = \pi_t^w W_t$$

$$L_{k,t} = \int_0^1 z_i h_{ikt} di$$

$$\frac{L_{k,t}}{L_t} = \left(\frac{W_t}{W_{k,t}}\right)^{\varepsilon_\ell}$$

Where the third equation follows from the first-order conditions from the households.

The HJB is then (suppressing the value function's arguments for brevity)

$$\rho J_t^w = \left[\int\int \left\{\frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma} - \frac{h(a,z;W_{k,t})^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right\}\mu_t(a,z)da\ dz - \frac{\theta_w}{2}(\pi_{k,t}^w)^2\right] + \frac{\partial J_t^w}{\partial W_{k,t}}\pi_t^w W_{k,t} + \frac{\partial J_t^w}{\partial t}$$

The FOC for wage inflation is then

$$-\theta_w \pi_{k,t}^w + \frac{\partial J^w(W_{k,t})}{\partial W_{k,t}} W_{k,t} = 0$$

$$\Rightarrow \quad \frac{\partial J^w(W_{k,t})}{\partial W_{k,t}} = \theta_w \frac{\pi_{k,t}^w}{W_{k,t}}$$

Taking the total differential of the marginal value of wages,

$$d\left(\frac{\partial J_t^w(W_{k,t})}{\partial W_{k,t}}\right) = \partial_{W_{k,t}}^2 J_t^w dW_{k,t} + \partial_{W_{k,t}}\partial_t J_t^w dt$$

and doing the same to the LHS of the wage inflation FOC,

$$d\left(\theta_w \frac{\pi_t^w}{W_{k,t}}\right) = \frac{\theta_w}{W_{k,t}} d\pi_t^w - \frac{\theta_w \pi_t^w}{W_{k,t}^2} dW_{k,t}$$

I can equate the two:

$$\frac{\theta_w}{W_{k,t}} d\pi_t^w - \frac{\theta_w \pi_t^w}{W_{k,t}^2} dW_{k,t} = \partial_{W_{k,t}}^2 J^w dW_{k,t} + \partial_t \partial_{W_{k,t}} J_t^w dt.$$

Taking expectations and dividing by dt yields

$$\frac{\theta_w}{W_{k,t}} \frac{\mathbb{E}_t[d\pi_t^w]}{dt} - \frac{\theta_w \pi_t^w}{W_{k,t}} \underbrace{\frac{dW_{k,t}}{dt}\frac{1}{W_{k,t}}}_{\pi_{k,t}^w} = \partial_{W_{k,t}}^2 J_t^w \frac{dW_{k,t}}{dt} + \partial_{W_{k,t}}\partial_t J_t^w$$

such that

$$\frac{\theta_w}{W_{k,t}} \frac{\mathbb{E}_t[d\pi_t^w]}{dt} - \frac{\theta_w \pi_t^w}{W_{k,t}} \pi_t^w = \partial^2_{W_{k,t}} J^w \pi_t^w W_{tk} + \partial_{W_{k,t}} \partial_t J_t^w \tag{D.3}$$

Next, the Envelope condition stipulates that

$$\rho \partial_{W_{k,t}} J_t^w = \left[\int \int \partial_{W_{k,t}} \left\{ \frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma} - \frac{h(a,z;W_{k,t})^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\} \mu_t(a,z) da\ dz \right]$$
$$+ \partial^2_{W_{k,t}} J^w \pi_t^w W_{k,t} + \partial_{W_{k,t}} J^w (W_{k,t}) \pi_t^w + \partial_{W_{k,t}} \partial_t J_t^w$$

Substituting in (D.3),

$$\rho \partial_{W_{k,t}} J_t^w = \left[\int \int \partial_{W_{k,t}} \left\{ \frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma} - \frac{h(a,z;W_{k,t})^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\} \mu_t(a,z) da\ dz \right]$$
$$+ \partial_{W_{k,t}} J_t^w (W_{k,t}) \pi_t^w + \frac{\theta_w}{W_{k,t}} \frac{\mathbb{E}_t[d\pi_t^w]}{dt} - \frac{\theta_w \pi_t^w}{W_{k,t}} \pi_t^w$$

and then the FOC,

$$\rho \theta_w \frac{\pi_{k,t}^w}{W_{k,t}} = \left[\int \int \partial_{W_{k,t}} \left\{ \frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma} - \frac{h(a,z;W_{k,t})^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\} \mu_t(a,z) da\ dz \right]$$
$$+ \theta_w \frac{\pi_{k,t}^w}{W_{k,t}} \pi_t^w + \frac{\theta_w}{W_{k,t}} \frac{\mathbb{E}_t[d\pi_t^w]}{dt} - \frac{\theta_w \pi_t^w}{W_{k,t}} \pi_t^w$$

it follows that

$$\rho \pi_{k,t}^w = \frac{W_{k,t}}{\theta_w} \left[\int \int \partial_{W_{k,t}} \left\{ \frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma} - \frac{h(a,z;W_{k,t})^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\} \mu_t(a,z) da\ dz \right] + \frac{\mathbb{E}_t[d\pi_t^w]}{dt}. \tag{D.4}$$

From the households' envelope condition, the change in utility from wages will be equal to the marginal utility, times the change in earnings:

$$\partial_{W_{k,t}} \left\{ \frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma} \right\} = c(a,z)^{-\gamma}(1-\tau) \partial_{W_{k,t}} \left(z \frac{W_{k,t}}{P_t} h(a,z) \right)$$

Where if households uniformly supply their labor to union k, and unions internalize their labor's demand:

$$h_{ikt}(a,z) = \frac{1}{Z} L_{k,t} = \frac{1}{Z} \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell} L_t$$

$$\Rightarrow \quad \partial_{W_{k,t}} \left\{ \frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma} \right\} = c(a,z)^{-\gamma}(1-\tau)\partial_{W_{k,t}} \left(z\frac{W_{k,t}}{P_t} \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell} \frac{1}{Z} L_t \right)$$
$$= c(a,z)^{-\gamma}(1-\tau)(1-\varepsilon_\ell)\frac{z}{Z}\frac{1}{W_{k,t}} \left(\frac{W_{k,t}}{P_t} \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell} L_t \right)$$
$$= c(a,z)^{-\gamma}(1-\tau)(1-\varepsilon_\ell)\frac{z}{Z}\frac{1}{P_t} L_{k,t}$$

For the effect of wages on labor disutility, I can directly evaluate

$$\partial_{W_{k,t}} h(a,z) = \frac{1}{Z}\partial_{W_{k,t}} \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell} L_t = -\varepsilon_\ell \frac{1}{Z}\frac{1}{W_{k,t}} \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell} L_t = -\varepsilon_\ell \frac{1}{Z}\frac{L_{k,t}}{W_{k,t}}$$

Plugging in the results into (D.4),

$$\rho\pi^w_{k,t} = \frac{W_{k,t}}{\theta_w} \left[\int\int \left\{ c(a,z)^{-\gamma}(1-\tau)(1-\varepsilon_\ell)\frac{z}{Z}\frac{1}{P_t}L_{k,t} + \frac{1}{Z}h(a,z)^{\frac{1}{\eta}}\varepsilon_\ell \frac{L_{k,t}}{W_{k,t}} \right\} \mu_t(a,z)da\ dz \right] + \frac{\mathbb{E}_t[d\pi^w_t]}{dt}$$

$$\rho\pi^w_{k,t} = \frac{\varepsilon_\ell}{\theta_w}\frac{L_{k,t}}{Z} \int\int \left\{ h(a,z)^{\frac{1}{\eta}} - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)z\frac{W_{k,t}}{P_t}c(a,z)^{-\gamma} \right\} \mu_t(a,z)da\ dz + \frac{\mathbb{E}_t[d\pi^w_t]}{dt}$$

Leading to the wage Phillips Curve

$$\frac{\mathbb{E}_t[d\pi^w_t]}{dt} = \rho\pi^w_t - \frac{\varepsilon_\ell}{\theta_w}\frac{L_t}{Z} \int\int \left(h(a,z)^{\frac{1}{\eta}} - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)zw_t c(a,z)^{-\gamma} \right) \mu_t(a,z)\ da\ dz \qquad \text{(D.5)}$$

where $w_t \equiv \frac{W_{k,t}}{P_t}$ is the real wage in the symmetric equilibrium where $W_{k,t} = W_t\ \forall k \in [0,1]$.

Log-linearizing for a representative agent,

$$\frac{\mathbb{E}_t[d\pi^w_t]}{dt} = \rho\pi^w_t - \frac{\varepsilon_\ell}{\theta_w}\frac{Y_t}{Z} \left[\left(\frac{Y_t}{Z} \right)^{\frac{1}{\eta}} - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)ZwY_t^{-\gamma} \right] \qquad \text{(D.6)}$$

$$\frac{\mathbb{E}_t[d\pi^w_t]}{dt} = \rho\pi^w_t - \frac{\varepsilon_\ell}{\theta_w}\frac{Y(1+\widehat{Y}_t)}{Z} \left[\left(\frac{Y}{Z} \right)^{\frac{1}{\eta}} (1 + \frac{1}{\eta}\widehat{Y}_t) - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)ZwY^{-\gamma}(1-\gamma\widehat{Y}_t) \right] \qquad \text{(D.7)}$$

$$\frac{\mathbb{E}_t[d\pi^w_t]}{dt} = \rho\pi^w_t - \frac{\varepsilon_\ell}{\theta_w}\frac{Y(1+\widehat{Y}_t)}{Z} \left[\left(\frac{Y}{Z} \right)^{\frac{1}{\eta}} \frac{1}{\eta}\widehat{Y}_t + \frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)Zw\gamma Y^{-\gamma}\widehat{Y}_t \right] \qquad \text{(D.8)}$$

$$\frac{\mathbb{E}_t[d\pi^w_t]}{dt} = \rho\pi^w_t - \frac{\varepsilon_\ell}{\theta_w}\frac{Y}{Z} \left[\left(\frac{Y}{Z} \right)^{\frac{1}{\eta}} \frac{1}{\eta} + \gamma\frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)ZwY^{-\gamma} \right] \widehat{Y}_t \qquad \text{(D.9)}$$

Note that this implies a Phillips Curve slope of the NKPC with respect to output is roughly 0.275, given the proposed parameters. If the slope is measured as just the component that relates to increases in marginal labor disutility, however, the slope is $\frac{\varepsilon_\ell}{\theta_w}\frac{Y_t}{Z}\left(\frac{Y_t}{Z}\right)^{\frac{1}{\eta}} = 0.07$.

Appendix E. Robustness

Appendix E.1. The Slope of the Phillips Curve

In the main parameterization, I set $\frac{\varepsilon}{\theta_\pi} = 0.10$, where $\theta_\pi = 100$. Mapping this to a discrete-time quarterly Calvo pricing model, if α is the percentage of unions that do not adjust their prices,

$$\frac{(1-\alpha)(1-\alpha e^{-\rho})}{\alpha} = 0.10 \;\Rightarrow\; \alpha = 0.74$$

such that roughly 26% of wage contracts reset every quarter and the average wage contract resets in slightly under a year.

Below in Figure E.8, I plot the cumulative impulse responses in the active fiscal/passive monetary regime under different parameterizations with different degrees of nominal rigidity. The main calibration, $\theta_\pi = 100$, is plotted with a solid line. Decreasing θ_π to 50 amounts to lowering nominal rigidities and doubling the slope of the Phillips curve, while doubling it to 200 is tantamount to halving the curve's slope.

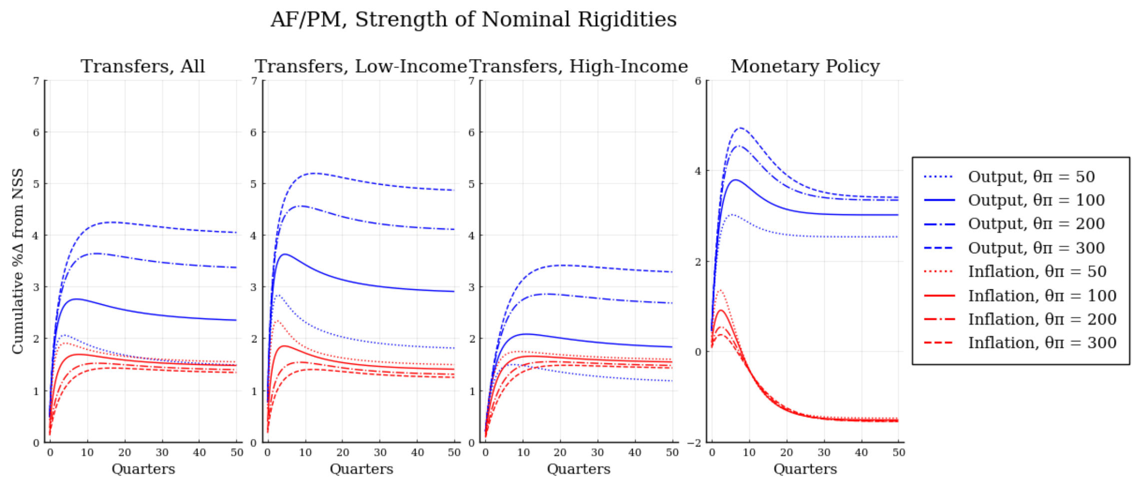


Figure E.8: Cumulative impulse response of output and inflation with different degrees of nominal rigidity. $\theta_\pi = 100$ is the baseline specification. Doubling θ_π halves the slope of the Phillips curve.

More nominal rigidities (and a flatter Phillips Curve) amplify the output response and smooth the path of inflation, while decreasing nominal rigidities does the converse. Even so, the price level eventually settles to roughly the same value in each experiment and parameterization. The ordering of the fiscal responses is also unchanged, even though their magnitudes are altered.

The implied cumulative sacrifice ratios (cumulative output gaps as a percentage of annual GDP divided by cumulative inflation) are plotted in Figure E.9. Lowering nominal rigidities compresses the difference in sacrifice ratios across policies, while increasing them increases the dispersion. Even

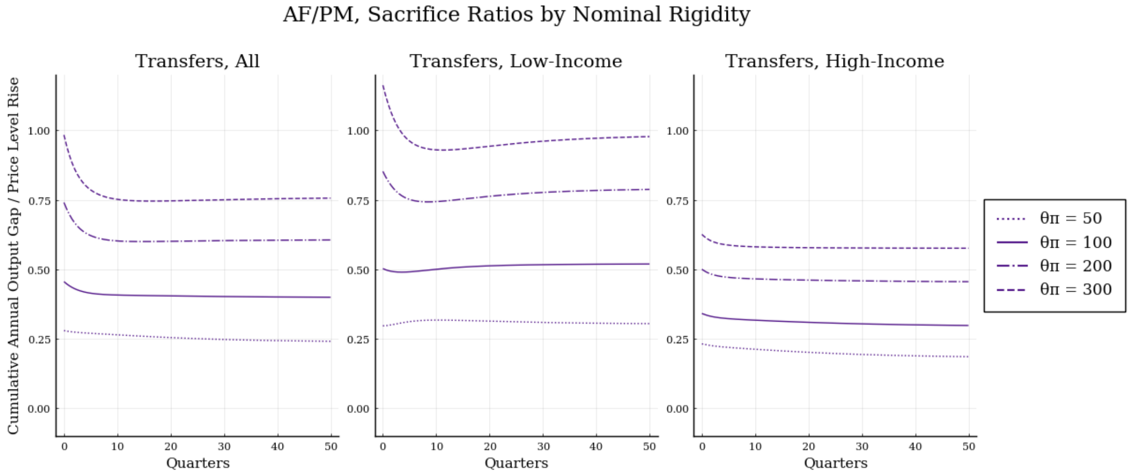


Figure E.9: Caption

so, the relative ordering between policies with the same parameterization is unchanged. Sacrifice ratios are lower for GDP changes induced by reductions in transfers to above-average income households, and are higher transfers to those with below-average income are reduced.

Appendix E.2. Firm Profits

In previous drafts of this paper, I included specifications where intermediate firms had a constant markup of $\varepsilon/(\varepsilon-1)$, where profits were distributed proportionally to labor income z. In my main specification, I set $\varepsilon \to \infty$, effectively making the intermediate firm sector perfectly competitive and removing firm profits from the model entirely.

In many HANK models, the distribution and cyclicality of firm profits can substantially affect the simulated dynamics. However, the inclusion or exclusion of these profits has little effect on my paper's conclusions. I plot cumulative impulse response functions for the active fiscal/passive monetary regime in Figure E.10, varying the elasticity of substitution of the output of intermediate firms ε as I do so. $\varepsilon = 7$ corresponds to profits composing 14% of national income, while $\varepsilon = 20$ reduces them to 5%. The main calibration of $\varepsilon \to \infty$ is plotted with solid lines.

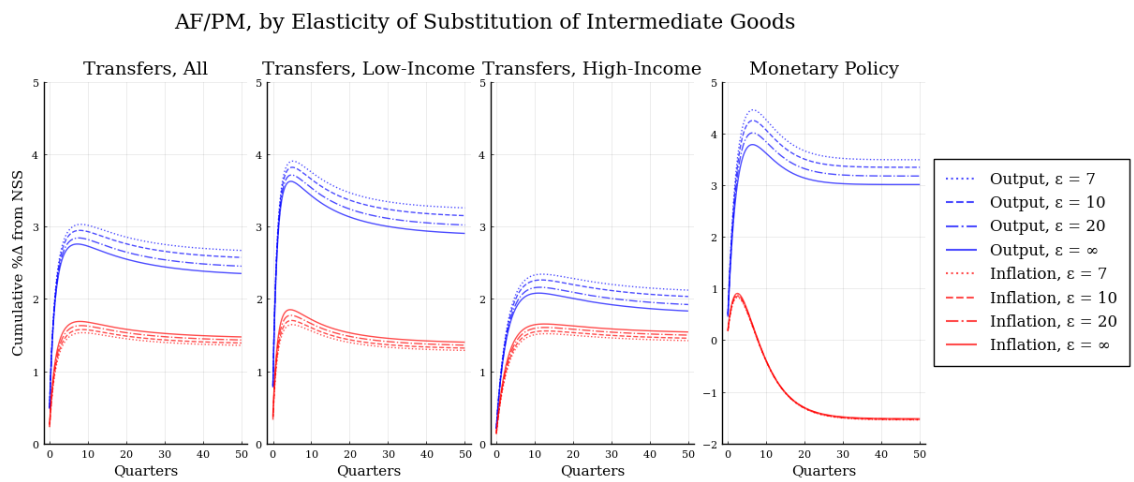


Figure E.10: Model solutions with different shares of profit income, $1/\varepsilon$.

Since I use a sticky-wage model with perfectly flexible output prices, however, firm markups are completely acyclic. Profit income thus only fluctuates due to changes in aggregate output, which are small when multiplied by the profit share. This profit income is further distributed proportionally to z to on-average wealthier agents with lower MPCs, so its effect on the model dynamics is small even when the profit share is realistically calibrated. As such, to avoid questions of the distribution of profits, I drop them from the model entirely.

Appendix F. Solving Bayer and Luetticke (2020) in Continuous Time

This section is best viewed after having already read Achdou et al. (2021), Ahn et al. (2018), and particularly Bayer and Luetticke (2020) as background; the below section largely amounts to a brief sketch of adapting Bayer and Luetticke (2020) to continuous time. For notational brevity, I write the infinitessimal generator operator of the concentrated Hamilton Jacobi Bellman equation as

$$\begin{aligned}\mathcal{D}[V] =& \lim_{t\downarrow 0} \frac{\mathbb{E}_t^{a,z}[V_t(a_{t+dt}, z_{t+dt})] - V_t(a_t, z_t)}{dt} \\ =& \frac{\partial V_t}{\partial a}(a,z)\frac{q_{NSS}}{q_t}\left[(1-\tau)w_t z h_t(a,z) + M_t(z_t;\zeta_t) - c + \left(r_t - \frac{dq_t}{dt}\frac{1}{q_t}\right)\frac{q_t}{q_{NSS}}a\right] \\ &+ \frac{\partial V_t}{\partial z}(a,z;\mu,\zeta)z\left[\frac{1}{2}\sigma_z^2 - \theta_z \log(z)\right]\end{aligned}$$

where the expectation operator is taken with respect to only the idiosyncratic variables. As in Achdou et al. (2021), I write the adjoint operator (which describes the Kolmogorov forward equation of the idiosyncratic state distribution) as $\mathcal{D}^*$, where the KFE operator is the adjoint of the maximized HJB operator in L^2 space. Additionally, I write expectation errors for a jump variable "J" as $d\delta_{J,t}$, such that $d\delta_{J,t} = dJ_t - \mathbb{E}_t[dJ_t]$.

Suppose aggregate shocks in the economy evolve according to

$$d\zeta_t = -\Theta_\zeta \zeta_t dt + d\epsilon_{\zeta,t}. \tag{F.1}$$

A sequential equilibrium following a perturbation from the steady state $W_{\zeta,0}$ is a resulting path of aggregate shocks $\{\zeta_t\}_{t\geq 0}$, a series of value functions $\{V_t(a,z)\}_{t\geq 0}$, consumption decisions and labor allocations $\{c_t(a,z), h_t(a,z)\}_{t\geq 0}$, distributions $\{\mu_t(a,z)\}_{t\geq 0}$, outstanding government debt $\{B_t\}_{t\geq 0}$, wages $\{w_t\}_{t\geq 0}$, nominal and real interest rates $\{i_t, r_t\}_{t\geq 0}$, bond prices $\{q_t\}_{t\geq 0}$, and inflation rates $\{\pi_t\}_{t\geq 0}$ where

$$dV_t(a,z) = \left\{\rho V_t(a,z) - \left[u(c_t(a,z)) - v(h_t(a,z)) + \mathcal{D}[V]\right]\right\}dt - \frac{\partial V_t(a,z)}{\partial a}d\delta_{qB,t} + d\delta_{V(a,z),t} \tag{F.2}$$

and if $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$, it follows that the FOC for consumption is

$$c_t(a,z)^{-\gamma} = \frac{\partial V_t}{\partial a}(a,z). \tag{F.3}$$

The distribution evolves according to

$$d\mu_t(a,z) = \mathcal{D}^*[\mu]dt \tag{F.4}$$

while labor is supplied to meet market demand:

$$h_t(a,z) = \frac{L_t}{Z} \tag{F.5}$$

Inflation is equal to nominal wage inflation, which follows the labor market Phillips Curve

$$d\pi_t^w = \left\{\rho\pi_t^w - \frac{\varepsilon_\ell}{\theta_w}L_t \int\int \left(v'(h(a,z)) - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)zw_t u'(c(a,z))\right) da\ dz\right\} dt + d\delta_{\pi^w,t} \tag{F.6}$$

Real wages are then constant:

$$w_t = w_{\text{NSS}} = \frac{\varepsilon - 1}{\varepsilon} \tag{F.7}$$

where ε is the elasticity of substitution between goods in the output sector; the profit-free version of the model sets $\varepsilon \to \infty$. The government's budget constraint must satisfy

$$dB_t = -(T_t - G_t)dt + r_t B_t dt + \frac{d\delta_{qB,t}}{q_t} B_t \tag{F.8}$$

where nominal bond prices and equity prices satisfy

$$dq_t = q_t\left(i_t + \omega - \frac{\omega}{q_t}\right) dt + d\delta_{q,t} \tag{F.9}$$

Equilibrium must also be consistent with the Fisher equation, the marginal cost equation, and the profit equation:

$$r_t = i_t - \pi_t \tag{F.10}$$

$$m_t = w_t \tag{F.11}$$

$$\Pi_t = [1 - m_t]Y_t \tag{F.12}$$

All goods consumed must be produced:

$$Y_t = L_t \tag{F.13}$$

and the idiosyncratic variables must aggregate:

$$C_t = \int_0^\infty \int_{\underline{a}}^\infty c_t(a,z)\mu_t(a,z)da\ dz \tag{F.14}$$

$$L_t = \int_0^\infty \int_{\underline{a}}^\infty zh_t(a,z)\mu_t(a,z)da\ dz \tag{F.15}$$

Finally, goods and financial markets must clear:

$$Y_t = C_t \tag{F.16}$$

$$B_t = \int_0^\infty \int_{\underline{a}}^\infty a\mu_t(a,z)da\ dz \tag{F.17}$$

I write the vector of forward-looking control variables as

$$X_{C,t} = (V_t(a,z),\ \pi_t,\ q_t)',$$

the set of state variables as

$$X_{1,t} = (\mu_t(a,z),\ B_t, \zeta_t)',$$

and the vector of static constraints as

$$X_{L,t} = (Y_t,\ L_t)',$$

(where many of the static constraints like the Fisher equation and the employment rules can be re-written to solve out the other static variables from the model). Stacking the controls, states, and static variables, I write

$$X_t = (X_{C,t}, X_{1,t}, X_{L,t})'$$

where dX_t represents the differentials of X_t. Using this succinct notation, the entire system (F.1-F.17) can be written as

$$\Gamma_0 dX_t = \Omega(X_t, d\delta_{X,t}, d\epsilon_{\zeta,t}) \tag{F.18}$$

where the rows of Γ_0 corresponding to static constraints are equal to zero.

I discretize the partial differential equations on the computer in the non-stochastic steady state where $X_t = X_{NSS}$, $dX_t = 0$, $d\delta_{X,t} = 0$, and $d\epsilon_{\zeta,t} = 0$, using the finite-differences methodology described in Achdou et al. (2021). This entails discretizing (F.18) via an upwind finite difference approximation for the partial derivatives along an asset grid (which I index by $i \in I \equiv \{1, \ldots, N_a\}$) and an income grid (which I index by $j \in J \equiv \{1 \ldots, N_z\}$). The tensor $V_{i,j,nss}$ then approximates the value function $V_{NSS}(a_i, z_j)$ in the discretized state space, while the tensor $\mu_{i,j,nss}$ approximates the distribution $\mu_{NSS}(a_i, z_j)$.

Before proceeding, I find it useful to define $\widehat{X}_t \equiv X_t - X_{NSS}$ as either the level deviations or the log deviations of the variables from their values in the non-stochastic steady state. As such, the complete system can be rewritten to become

$$\Gamma_0 d\widehat{X}_t = \widehat{\Omega}(\widehat{X}_t, d\delta_{X,t}, d\epsilon_{\zeta,t}) \tag{F.19}$$

where the arguments are the deviation terms. The steady state thus satisfies $\widehat{\Omega}(\mathbf{0}) = \mathbf{0}$. I then proceed to solve for the dynamics of the economy following aggregate shocks. Practically, the dimensionality of the discretized value functions and distributions necessitate dimension reduction. However, for clarity, I first describe the process *without* dimension reduction.

Appendix F.1. Without Dimension Reduction

With the non-stochastic steady state (NSS) in hand, I then calculate the numerical Jacobian of the system at the NSS using automatic differentiation. Differentiating the entire system with respect to just the arguments in X_t alone, I can write the Jacobian of the system with respect to its X_t variables at the non-stochastic steady state as

$$\Gamma_{X,X} \equiv \nabla_X \widehat{\Omega}(\mathbf{0})$$

While the derivatives of the system with respect to the expectation errors and the perturbations are

$$\Gamma_{X,\delta} \equiv \nabla_{d\delta} \Omega(\mathbf{0})$$

$$\Gamma_{X,W} \equiv \nabla_{dW_\zeta} \Omega(\mathbf{0})$$

A first-order Taylor expansion of the system around the steady state without any shocks (and where $d\widehat{X}_t = 0$) is then

$$\Gamma_0 d\widehat{X}_t = \Gamma_{X,X} \widehat{X}_t dt + \Gamma_{X,\delta} d\delta_{X,t} + \Gamma_{X,W} d\epsilon_{\zeta,t} + \mathcal{O}(\|\widehat{X}_t, d\delta_{X,t}, d\epsilon_{\zeta,t}\|^2)$$

I then solve

$$\Gamma_0 d\widehat{X}_t = \Gamma_{X,X}\widehat{X}_t dt + \Gamma_{X,\delta} d\delta_{X,t} + \Gamma_{X,W} d\epsilon_{\zeta,t} \tag{F.20}$$

using the generalized eigenvalue methodology described in Sims (2002). If the system has more stable generalized eigenvalues than it has control variables, the dimensionality of the linear subspace being used to approximate the system's stable manifold is too large to ensure that the dynamics are unique, such that multiple equilibria are possible (sunspots). If the system has fewer stable eigenvalues than state variables, then the equilibrium cannot exist. I verify that the number of stable eigenvalues in my system matches the number of state variables, such that the solution exists and is unique.

While straightforward, this approach is too computationally costly to be feasible with the number of gridpoints that I employ to solve my full model. As such, I use the dimension reduction strategy of Bayer and Luetticke (2020) before calculating the Jacobian of (F.19).

Appendix F.2. With Dimension Reduction

I write the 2-dimensional discrete cosine transform (DCT) of a 2-dimensional array A as $\theta^A = \text{DCT}(A)$, where its inverse $\text{DCT}^{-1}(\theta^A) = A$. I can write the transformation of the value function in the non-stochastic steady state as

$$\{\theta^V_{(i,j),nss}\}_{(i,j)\in I\times J} = \text{DCT}(\{V_{(i,j),nss}\}_{(i,j)\in I\times J})$$

I then compute the "energy" (to use the terminology of Bayer and Luetticke (2020)) of the $\theta^V_{i,j,nss}$ coefficients as

$$E_{ij} = \frac{[\theta^V_{(i,j),nss}]^2}{\sum_{(i,j)\in I\times J}[\theta^V_{(i,j),nss}]^2}$$

Sorting the coefficients by their energy from greatest to least, I then identify those coefficients that contain a cumulative $1-\kappa$ share of the coefficients' energy, where κ is a small number. I label the set of these coefficients (which are effectively the ones with the largest absolute value) as Θ_E; these coefficients explain most of the variation of the value function in the steady state.

As in Bayer and Luetticke (2020), I then move toward constructing a perturbation solution of the equilibrium system, but perturbing only high-energy coefficients in Θ_E. Otherwise, I keep the lower-energy coefficients constant, at their steady state values:

$$\tilde{\theta}^V_{i,j,t} = \theta^V_{(i,j),t} + \mathbf{1}_{\{(i,j)\in\Theta_E\}}\widehat{\theta}^V_{(i,j),t}$$

where $\widehat{\theta}^V_{i,j,t}$ is the coefficient's deviation at time t from its NSS value.

The DCT is a linear operator. As such, I can write the differentials of the coefficients as

$$\{d\theta^V_{(i,j),t}\}_{(i,j)\in I\times J} = d\left[\text{DCT}(\{V_{(i,j),t}\}_{(i,j)\in I\times J})\right] = \{d\theta^V_{(i,j),nss}\}_{(i,j)\in I\times J} = \left[\text{DCT}(\{dV_{(i,j),nss}\}_{(i,j)\in I\times J})\right]$$

and similarly I write

$$d\tilde{\theta}^V_{(i,j),t} = \mathbf{1}_{\{(i,j)\in\Theta_E\}} d\theta^V_{(i,j),t}$$

By perturbing only the $|\Theta_E|$ largest-magnitude coefficients instead of the full $N_a \times N_z$ elements of the discretized value function, I can greatly reduce the dimensionality of the problem. Of course, this only reduces the number of control variables. To reduce the number of state variables in the distribution, I also employ the fixed copula transformation of Bayer and Luetticke (2020).

I write the discretized joint cumulative distribution function $F_{\mu(a_i,z_j)}$, and the marginal CDFs as $F_{\mu(a_i)}$ and $F_{\mu(z_j)}$. The copula is then the joint distribution interpolated onto the marginal ones:

$$\text{Cop} = \text{Interp}(\{F_{\mu(a_i,z_j),nss}\}_{ij}, \{F_{\mu(a_i),nss}\}_i, F_{\mu(z_j),nss}\}_j)$$

where the nss subscript denotes the steady state values. It then follows that $\text{Cop} : [0,1]\times[0,1] \to [0,1]$ maps cumulative marginal distributions to a joint distribution, as predicted by the rank correlations of the steady state. Outside of the steady state, I then approximate the joint cumulative distribution $F_{\mu(a_i,z_j),t}$ at time t as

$$F_{\mu(a_i,z_j),t} \approx \text{Cop}(F_{\mu(a_i),t}, F_{\mu(z_j),t}),$$

from which the marginal joint *density* function μ_{ij} may be derived. Using this object, I can then iterate the Kolmogorov Forward Equation to obtain $d\mu_{ij}$, which can be integrated (or summed, since the functions are discretized) to obtain the evolution of the differentials

$$\{(dF_{\mu(a_i),t}, dF_{\mu(z_j),t})\}_{ij}.$$

As Bayer and Luetticke (2020) note, this approximation allows me to track only the N_a and N_z dimensional marginal CDFs instead of their joint one to describe the economy, so long as the rank correlations outside of the steady state are similar to those represented in the steady state (which Bayer and Luetticke (2020) show is generally the case in Bewley-Aiyagari models).

I then define the dimension-reduced set of controls as

$$\tilde{X}_{C,t} = (\{\tilde{\theta}^V_{i,j,t}\}_{(i,j)\in\Theta_E},\ \pi_t,\ \pi^w_t,\ q_t)'$$

and the dimension-reduced set of states as

$$\tilde{X}_{1,t} = (\{F_{\mu(a_i),t}\}_i, \{F_{\mu(z_j),t}\}_j,\ B_t,\ w_t, \zeta_t)',$$

Once again stacking the reduced controls, states, and static variables, I write

$$\tilde{X}_t = (\tilde{X}_{C,t}, \tilde{X}_{1,t}, X_{L,t})'$$

and the system (F.18) is approximated by a smaller one:

$$\tilde{\Gamma}_0 d\tilde{X}_t = \tilde{\Omega}(\tilde{X}_t, d\delta_{X,t}, d\epsilon_{\zeta,t})$$

where $\tilde{\Omega}$ calculates the value function and joint distribution given the DCT coefficients and the marginal distribution, feeds them back into the original Ω function, and then from there recovers the resulting truncated DCT coefficients and marginal CDFs' time differentials. Just like before, this system can also be written in terms of just the differences (or log differences) of the variables from their non-stochastic steady state values. The rest of the linearization steps and solution methods then proceed exactly in the same manner as they do in the version without dimension reduction, as reviewed in the prior subsection of this appendix.

I solve the model over a uniform grid of $N_a = 100$ points spaced nonlinearly from 0 to 60, with more grid points at the bottom of the asset distribution. I use $N_z = 50$ grid points from 0.01 to 5.5.