Active vs. Passive Policy and the Trade-Off Between Output and Inflation in HANK

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Abstract

When fiscal policy is active and monetary policy is passive in a heterogeneous agent New Keynesian (HANK) model, deficit-financed transfers to low-asset households lead to similar cumulative inflation but greater increases in real output than transfers to wealthier households. I use the inverse of the "Phillips multiplier," the price level sacrifice ratio, to quantify this dynamic. Household heterogeneity and targeted policy change the timing of output gaps, making this consistent with the Phillips Curve and rendering conventional sacrifice ratio intuition misleading for assessing the inflation/output trade-off between policies.

Keywords: fiscal theory, heterogeneity, inflation, HANK

JEL: E63, E31, E12

1. Introduction

The trade-off between real output and inflation following unanticipated changes to fiscal policy remains a long-standing open question in macroeconomics. In the parlance of Leeper (1991), much of the previous literature has focused on models where monetary policy is "active" and fiscal policy is "passive." However, monetary policy in the United States has been constrained by the zero lower bound for much of the early 21st century, while fiscal authorities have increasingly responded to changing macroeconomic conditions with tax cuts and transfer programs. As such, this paper departs from standard policy regime assumptions

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and instead explores the implications of active fiscal and passive monetary policy for output and inflation in a canonical heterogeneous agent New Keynesian (HANK) model with idiosyncratic income risk and incomplete asset markets, such that households are heterogeneous in their marginal propensities to consume (MPCs).

If the government sends deficit-financed transfer payments to low-wealth households with high MPCs, then the cumulative increase in real GDP is predictably larger than when the transfers are sent to wealthier, lower-MPC households. However, the long-term effect on the price level – total cumulative inflation – is largely the same under both transfer policies provided they lead to similar amounts of nominal government debt (net nominal private assets) that are not paid off by future tax revenue and are instead inflated away. If all taxes and transfers are lump-sum and set exogenously by policy, the net present value of inflation is entirely invariant to a fiscal stimulus' targeting.

This finding runs counter to the intuition that output gaps and the price level might move proportionally as measured by stable "sacrifice ratios," which measure the cumulative percentage change in real GDP relative to trend associated with a one percentage point abatement in inflation. This quantity is closely related to the inverse slope of the New Keynesian Phillips curve, a ubiquitous feature in models with nominal rigidities. However, the total inflationary impact of a policy is not just the difference in the inflation rate before and after its implementation, but the total rise in the price level – the cumulative inflation – ascribed to the policy. As such, I instead use what I call the *price level sacrifice ratio*, defined as the ratio of cumulative annual output gaps to cumulative inflation, making it the theoretical inverse of the total "Phillips multiplier" empirically defined and estimated in Barnichon and Mesters (2021) and discussed in Lepetit and Furlanetto (2024) (but for fiscal policy rather than monetary policy). I show that for the New Keynesian Phillips Curve, the timing of the output gaps (an endogenous object) is crucial for determining the size of these sacrifice ratios as well. Fast expansions or contractions move inflation less overall than slower ones, even with the simplest New Keynesian Phillips Curve. How a policy interacts with MPC heterogeneity changes this timing, and thus changes the trade-off.

My analysis comes in two parts. First, I briefly outline a simplified two-agent New Keynesian (TANK) model where one group of households smooths consumption with their savings while the other group is constrained to spending their income as soon as it is received. I then provide closed-form analytical results for the simple economy to ascertain why heterogeneity is of only minor relevance for the determination of the overall price level but important for the output response, and how this is consistent with the Phillips Curve.

In the second part of my analysis, I replace the two-agent block of the TANK model with a calibrated distribution of households over asset and income states. Agents face uninsurable idiosyncratic income risk and incomplete markets, yielding a canonical HANK framework with active fiscal policy and passive monetary policy. Unlike in the TANK model, the distributions of assets and MPCs are endogenous and targeted to match empirical moments, while the income distribution is parameterized to match data measurements of earnings autocorrelation and volatility. Although more complicated than the TANK setting, this model delivers similar conclusions. However, the added realism of asset and income inequality, precautionary savings motives, and endogenous MPCs make the setting an ideal "laboratory" with which to examine how active fiscal policy regimes function when fiscal transfers are targeted to one group but not another.

I provide an overview of the literature in Section 2. In Section 3, I discuss the simple two-agent model. Section 4 outlines the quantitative HANK model and the numerical experiment. Section 5 describes the results of the HANK experiment, with a particular focus on the sacrifice ratios generated by each policy. Section 6 concludes.

2. Related Literature

This paper solves a calibrated incomplete markets business cycle model with New Keynesian nominal rigidities to evaluate the effects of fiscal policy when fiscal policy is active and monetary policy is passive. While textbook New Keynesian models initially emphasized monetary policy, incomplete markets' endogenous MPC heterogeneity has led many HANK papers to explore (passive) fiscal policy, either as a transmission method for monetary policy as in McKay et al. (2016) and Kaplan et al. (2018) or in its own right as in papers like Auclert et al. (2024) and Auclert et al. (2023b). Few papers, however, directly consider fiscal shocks that explicitly target one group of households or another – much less fiscal shocks that do so and are active. My paper helps fill this gap.

As alluded to in the introduction, I use the terms “active” and “passive” to describe fiscal and monetary policy in the style of Leeper (1991). When fiscal policy is “active,” it does not automatically stabilize the government’s real debt to steady state levels over time for all sequences of the price level, but instead lets the price level stabilize the real value of nominal liabilities via an immediate jump or inflation. To this end, bonds are nominally denominated in my model and monetary policy is “passive” and accomodative with inflation such that it raises nominal interest rates less than one-for-one with inflation, unlike in the aforementioned HANK papers McKay et al. (2016), Kaplan et al. (2018), and Auclert et al. (2024), which all feature active monetary policy and real bonds. My chosen active fiscal/passive monetary environment thereby echoes the fiscal theory of the price level (FTPL) mechanism described in Woodford (1995), Sims (1994, 2011), Cochrane (2001, 2018b,a, 2023), Bianchi et al. (2023), and many others.

Although my framework is conceptually similar to the FTPL literature (and FTPL describes my simple two-agent TANK model in Section 3), the introduction of incomplete markets results in nontrivial differences. Farmer and Zabczyk (2019) notes that FTPL can fail to deliver determinacy when the steady-state real interest rate is endogenous, a feature that Hagedorn (2024) shows to also be true for incomplete markets models. Instead, Hagedorn (2016, 2023, 2024) contend that if the government issues more nominal debt, then the price level adjusts to equate the real demand for those assets to their real supply. Indeed, Angeletos et al. (2024) consider a HANK model at a zero liquidity limit (in the style of Werning (2015) but with active fiscal policy) to show that HANK equilibria are determined by a failure of Ricardian equivalence. My paper concurs with this view; as in Angeletos et al. (2024) and Hagedorn (2024), in Appendix B I show that my HANK model remains determinate even when both fiscal and monetary policy are passive.

While most of the HANK literature has committed to exploring the conventional passive fiscal/active monetary setting with real bonds, Hagedorn et al. (2019) is an exception in that it examines the strength of fiscal policy with an alternative determinacy mechanism and nominal bonds. Specifically, Hagedorn et al. (2019) achieves determinacy in part via a fiscal commitment to a fixed (indeed constant) path of nominal government bonds, generating a kind of price level targeting result to stabilize real debt. Non-Ricardian agents and nominal government debt are also important in my setting. However, my model does not assume the government exogenously sets a path for nominal debt. Rather, it continues the precedent of previous FTPL papers: the government uses fiscal policy to commit to a sequence of real transfers by issuing nominal liabilities as required by the current price level, whatever the current price level may be. The price level must then adjust in equilibrium to bring the real value of these outstanding nominal liabilities back to their steady-state values to stay on the system's stable manifold – or else the non-Ricardian consumers would send the economy down an explosive path that cannot be an equilibrium.

My analysis is the first to study active fiscal policy with group-specific transfers in a fully-fledged HANK model with nominal rigidities, but Kaplan et al. (2023) similarly examine active fiscal/passive monetary policy in an endowment incomplete markets economy without nominal rigidities – and thus without a Phillips curve relating output gaps to changes in the price level. The authors' endowment economy experiments[1] show that fiscal transfers cause more short-run inflation than in a representative agent model by reallocating resources to constrained households. Over time, inflation converges to a one-time price level jump. Importantly, whether transfers are targeted or not has little impact on inflation dynamics, a result preserved in my model with sticky prices. While our focuses differ, my work complements theirs by emphasizing output-inflation trade offs in settings with nominal rigidities.

All of my simulations are for certainty-equivalent models using linearized perturbations from a non-stochastic steady state. I use the sequence-space Jacobian technique of Auclert et al. (2021) to solve the HANK model. I also use the state-space method of Bayer and Luetticke (2020) (a modification of Reiter (2009)) solved via a Schur decomposition as an added numerical determinacy check. All models are solved using finite difference approximations in continuous time. Werning (2015) and Acharya and Dogra (2020) show that the determinacy and dynamics in HANK models is strongly affected by the cyclicality of

[1] Kaplan et al. (2023) additionally find that while a heterogeneous agent active fiscal/passive monetary economy retains uniqueness and determinacy when the government runs surpluses in the steady state, multiple equilibria may emerge when the government runs perpetual deficits and $r < g$. The authors suggest policy rules for eliminating this multiplicity of equilibria and run most of their simulations in an $r < g$ setting, but I consign my model to a more theoretically conventional environment with positive steady state primary surpluses and $r > g$.

idiosyncratic income risk; I abstract away from these forces in my model by considering an environment where the risk is acyclical, bringing the model closer to the RANK-like benchmark obtained in Werning (2015).

3. Analytic Expressions and a Simple Model

Consider two policies that lead to the same net present value of government deficits, discounted at the steady-state discount rate. One policy sends transfers to households who spend the money immediately, while the other sends money to those who are forward-looking and save. Both policies lead to the same present value of inflation if inflation (accommodated by monetary policy) restores real government liabilities to their steady-state level; the net present value of inflation will be invariant (analytically, to first-order) to the distribution of transfer recipients if deficits are exogenous. If the rate of discounting is small, this present value of inflation is then (quantitatively) very close to the overall actual rise in the price level and the rise in nominal debt. Alternatively, if two policies lead to the exact same accumulation of long-term nominal debt, then the increase in the price level will also be exactly the same. In either case, the change in real GDP might vary between the two policies, as it strongly depends on the MPCs of the recipient households.

This phenomenon is analytically apparent in a TANK model where a measure $(1-\mu)$ of constant relative risk averse (CRRA) households ("savers") discount the future at a rate of ρ and choose their consumption via a standard Euler equation, while another measure μ of households are constrained to be hand-to-mouth ("spenders"). Aggregate demand is the weighted sum of the two households' consumption choices, while firms set prices according to a standard Phillips Curve. Suppose the government takes on debt to send transfers to one household or the other at time t and never raises taxes to pay this debt back, while the central bank accommodates this by fixing nominal interest rates to a constant value. Appendix A.1 characterizes this equilibrium with five equations.

3.1. Government Deficits and Inflation

When the government issues transfers to either group of households, it exogenously changes its total net primary surplus T_t and issues more nominal debt with a real value of B_t at a real interest rate of $r_t = i - \pi_t$:

$$\frac{dB_t}{dt} = -T_t + (i - \pi_t)B_t. \tag{1}$$

This equation is the same in HANK, RANK, and TANK. In Appendix A.3, I integrate the equation forward, apply the saver household's transversality condition and log-linearize to show

$$\mathbb{E}_t \int_t^{\infty} e^{-(\tau-t)r} \widehat{\pi}_\tau d\tau = \widehat{B}_t - \mathbb{E}_t \left[\frac{T}{B} \int_t^{\infty} e^{-(\tau-t)r} \widehat{T}_\tau d\tau \right] \tag{2}$$

where variables without a time subscript denote values in the non-stochastic steady state, while hatted variables denote percent deviations thereof.

Up to a first-order approximation, the present value of inflation (discounted according to the steady state discount rate, here ρ) will be equal to the discounted value of future unfunded deficits as a percentage of steady state debt – plus whatever excess real debt $\widehat{B}_t$ the government carried over into the period, which will be zero if the economy was in its non-stochastic steady-state prior to the shock.

If deficits $(-T_\tau)_{\tau \geq t}$ are themselves exogenous, then household heterogeneity does not enter into equation (2) at all, and so the present value of inflation discounted at the steady-state rate is invariant to household heterogeneity. As such, the only way that household heterogeneity can affect the total rise in the price level is by changing the *timing* of inflation while keeping its net present value constant. These timing effects are very small, however, if steady state interest rates are small ($r = 0.005$ in a quarterly calibration that targets 2% annual rates) and inflation mean reverts quickly after a few years.

By this logic, a surprise transfer to high MPC households might lead to slightly *less* long-run inflation than alternative transfer arrangements if inflation peaks rapidly following the transfer shock. This could occur if large immediate output gaps translate to more initial short-term inflation through the Phillips curve, reducing real interest rates for the government and thus the amount of interest expense that the economy must inflate away. Indeed, this is exactly what occurs the simulated HANK model. This also provides an opportunity to describe how nominal rigidities fit into this picture.

3.2. How is this consistent with the Phillips Curve?

Although the net present value of inflation is pinned down by policy, the exact path the price level takes is still described by the New Keynesian Phillips Curve:

$$\rho \pi_t = \frac{\mathbb{E}_t[d\pi_t]}{dt} + \nu \widehat{Y}_t \tag{3}$$

where π_t is the inflation rate, $\widehat{Y}_t$ is the output gap, and ν is the slope of the Phillips Curve.

The Phillips Curve does describe the dynamic relationship between inflation and the output. Differentiating inflation π_t by the present value of all future output gaps yields a conventional sacrifice ratio (change in the net present value of future output gaps per percentage point decline in inflation) of $1/\nu$. However, even in this very simplified model, the relationship between total *cumulative* inflation and output depends on the *timing* of the output gaps. In Appendix A.2, I show that integrating (3) forward twice yields

$$\mathbb{E}_t \int_t^\infty \pi_\tau d\tau = \nu \mathbb{E}_t \int_t^\infty (\tau - t) e^{-\rho(\tau - t)} \widehat{Y}_\tau d\tau \tag{4}$$

If the rate of discounting or amount of time since the shock has transpired is small, output gaps twice as far into the future count roughly double toward the total amount of inflation; the further the output gap is into the future, the more inflationary it is.

The intuition is straightforward. It is true that inflation at time t jumps higher when current and future output gaps jump higher, all else equal. However, if firms or workers and unions take time to adjust their prices, then they are limited in how much they can immediately raise their prices in response to an acute surge in output. Additionally, they are forward-looking, so past output and inflation are sunk; only future output gaps matter for how they set prices. If real GDP returns to its steady state value quickly, these future output gaps may be small, even if past output gaps have been large. In this sense, price-setters in the economy tend to fall "behind the curve" for the transitory-but-potent real GDP expansions that transfers to high hand-to-mouth agents generate. By the time the economy returns to steady state, cumulative real output can rise higher for the same rise in the price level when it rises faster.

3.3. Heterogeneity Affects Households and GDP

Although the present value of inflation is set by government policy, MPC heterogeneity in the TANK model leads the distribution of the transfers to strongly affects the path of output. In Appendix A.4.1, I show that the sequence of output gaps $(\widehat{Y}_t)_{t\geq 0}$ in the TANK model will satisfy an intertemporal Keynesian cross of the kind described by Auclert et al. (2024). Integrating the relationship forward, I show that transfers to spenders will have an additional contemporaneous multiplier effect on GDP of $1/(1-\mu)$ beyond what transfers to savers induce for any two policies that generate the same path of real debt.

3.4. Nominal Debt and the Price Level

Understanding the price level sacrifice ratio requires one to understand the price level's relationship with nominal debt. To say that the change in the price level is almost invariant to the targeting of active fiscal transfer payments is tantamount to stating that the eventual level of nominal debt generated is invariant to the transfers' targeting, so long as the real debt level is stationary – as it is in most models. To see this, suppose a general equilibrium model starts in steady-state. One can simply note that if B is the steady-state level of real debt, p_t is the price level, and B_t^n is the amount of nominal debt at time t, then

$$B = \frac{B_-^n}{p_-} = \lim_{t\to\infty} \frac{B_t^n}{p_t}$$

where B_-^n, p_- denote the nominal bonds in circulation and the price level before a transitory shock occurs.

The insight gained from stationary real debt is often limited because the evolution of nominal debt B_t^n is itself an endogenous object. This is the case in my model as well: unlike Hagedorn et al. (2019), the government in my model does *not* follow an exogenous nominal debt target to establish determinacy. However, if fiscal transfers are sent out with a slow-adjusting price level and nominal interest rates stay constant, nearly the same volume of nominal debt is taken on regardless of where the real transfers go – and so the price level responds nearly the same way in the long term. As I show in Section 5, if the shocks to transfers to different groups are chosen such that they all generate the same asymptotic nominal debt level, then they will all induce exactly the same change in the price level – essentially by construction – but potentially different contributions to the sequence of output gaps.

The fact that nominal debt is endogenous in the model also highlights why the long-term inflation dynamics can be more complicated in models where monetary policy is active. If the nominal interest rate adjusts in response to the state of the economy, then this also endogenously changes the nominal debt level induced by the policy, making the eventual price level interact with the Taylor rule and related endogenous variables. This interaction is absent under an interest rate peg with exogenous fiscal policy – making the long term rise in the price level insensitive to other features of the economy.

4. A Heterogeneous Agent New Keynesian (HANK) Model

Given the basic intuition of Section 2, I next solve a calibrated incomplete markets economy to verify that the insights from the stylized dynamics of the simple TANK model carry over into a more realistic HANK environment. The experiment is largely the same: I fix the present value of a surprise debt-funded active fiscal transfer and compare the rise in the price level and the cumulative rise in real GDP when transfers go out to richer agents who save versus poorer agents who spend. In the HANK model, however, transfers are distributed contingent on households' income, rather than a fixed "type" as in the TANK model.

Time $t \geq 0$ is continuous. At a high level, the economy is populated by households who have the same preferences, but face borrowing constraints and different paths of idiosyncratic labor income shocks that they cannot fully insure. These households save by holding long-lived nominal government bonds and supply their labor to the market via decentralized unions. The output sector is perfectly competitive; wages adjust with nominal rigidities, such that labor demand and output are demand-determined. The government issues debt to pay for transfer payments and does not necessarily raise taxes to keep the debt from growing exponentially. A central bank sets nominal interest rates according to either a Taylor rule or an interest rate peg. The numerical solutions are all for a perfect foresight environment linearized around a nonstochastic steady state (NSS); once the shock is realized, the transition dynamics are deterministic and known to the agents in the model.

4.1. Households

A measure 1 continuum of households inhabit a Bewley-Aiyagari setting where they have two dimensions of ex-post heterogeneity: their labor-augmenting productivity z (generating income inequality) and their real asset position a (which agents endogenously determine based on their consumption choices). The setting has become the workhorse for single-asset HANK models like that of McKay et al. (2016). Households choose their consumption choice c with an intertemporal elasticity of substitution of $1/\gamma$ and supply hours worked h according to a rule set by unions, in so doing incurring labor disutility with a Frisch elasticity of η. The government taxes labor income at a fixed rate of τ. If $V_t(a, z)$ is a household's value function at time 0 given their asset position a at steady state bond prices and labor productivity z, the household problem is

$$
\begin{aligned}
V_0(a_0, z_0) = \max_{\{c_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{h_t(a_t, z_t)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] dt \\
\text{s.t. } \frac{da_t}{dt} = (1-\tau) w_t z_t h_t(a_t, z_t) + r_t a_t + M_t(z_t; \zeta_t) - c_t \\
d\log(z_t) = -\theta_z \log(z_t) dt + \sigma_z dW_{t,z} \\
a_t \geq 0.
\end{aligned}
$$

Here, W_t is a classical Wiener process (Brownian motion), such that log labor income follows an Ornstein-Uhlenbeck process in the non-stochastic steady state that reverts to the mean at a rate of θ_z. The agents can receive transfers from the government $M_t(z_t, \zeta_t)$ that depend on their position in the income distribution and an aggregate fiscal shock ζ_t.

The household's problem can be recursively formulated as a Hamilton Jacobi Bellman (HJB) equation:

$$
\begin{aligned}
\rho V_t(a, z) = \max_c \Bigg\{ & \left[\frac{c^{1-\gamma}}{1-\gamma} - \frac{h_t(a, z)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \\
& + \frac{\partial V_t}{\partial a}(a, z) \left[(1-\tau) w_t z h_t(a, z) + M_t(z_t; \zeta_t) - c + r_t a \right] \\
& + \frac{\partial V_t}{\partial z}(a, z) z \left[\frac{1}{2}\sigma_z^2 - \theta_z \log(z) \right] + \frac{\partial^2 V_t}{\partial z^2}(a, z) \frac{1}{2} \sigma_z^2 z^2 + \frac{\partial V_t}{\partial t}(a, z) \Bigg\}
\end{aligned} \tag{5}
$$

where households take the path of prices w and r as given and subsumed into the time subscript of the value functions.

The distribution of households over idiosyncratic states is $\mu_t(a,z)$; it evolves according to the standard Kolmogorov Forward Equation (KFE)

$$\frac{\partial \mu_t}{\partial t}(a,z) = -\frac{\partial}{\partial a}\left(\frac{da_t}{dt}\mu_t(a,z)\right) - \frac{\partial}{\partial z}\left(\frac{\mathbb{E}_t[dz_t]}{dt}\mu_t(a,z)\right) + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left(\sigma^2 z^2 \mu_t(a,z)\right). \tag{6}$$

4.2. Fiscal Policy

The model's fiscal authority collects aggregate taxes (net of transfers) equal to T_t; real government expenditures G_t are included in the following equations for generality but are set to be zero in equilibrium. The aggregate price level in the economy is p_t.

The market value of real debt outstanding is $B_t \equiv B_t^n/p_t$ and evolves according to backward-looking equation

$$\frac{dB_t}{dt} = -(T_t - G_t) + (i_t - \pi_t)\, B_t. \tag{7}$$

As a baseline, the fiscal authority in the model taxes labor income at a rate of τ, such that if total effective labor employment in the economy is L_t and real wages are w_t, total income taxes are $\tau w_t L_t$ per unit of time. Households also receive lump-sum transfers from the government, which aggregate to total lump-sum transfers M_t. Total tax revenue is

$$T_t = \tau w_t L_t - M_t. \tag{8}$$

In the nonstochastic steady state (NSS), the government balances its budget and rebates transfers uniformly such that $M_{NSS} = \tau w L_{NSS} - r_{NSS} B_{NSS}$.

Outside of the steady state, transfers can either be made to those below median z (denoted $z_{0.50}$), above median z, or to all households:

$$\begin{aligned} M_t(z,\zeta_t) = & 4Y_{NSS} \times \left(\zeta_{\mathrm{ALL},t} + \frac{1}{0.5}\mathbf{1}\{z \le z_{0.50}\}\zeta_{\mathrm{BELOW},t} + \frac{1}{0.5}\mathbf{1}\{z > z_{0.50}\}\zeta_{\mathrm{ABOVE},t}\right) \\ & - \kappa\,(B_t - B_{NSS}) \end{aligned} \tag{9}$$

where $\zeta_{ALL,t}, \zeta_{BELOW,t}, \zeta_{ABOVE,t}$ are aggregate shocks that follow (14). The transfer shocks are therefore scaled as a percentage of annual steady state GDP and are also scaled by the mass of the recipients to represent the same amount of aggregate transfer spending.

The last term regulates a fiscal rule that determines whether or not fiscal policy is active or passive. If $\kappa > r_{NSS}$, then taxes automatically adjust to bring debt back to its nonstochastic steady state, making fiscal policy passive. However, if $\kappa < r_{NSS}$, then inflation must stabilizes debt, making fiscal policy active. In the main text of this paper, I set $\kappa = 0$, rendering fiscal policy unambiguously active and only deviate from this assumption in Appendix B.

Total transfers aggregate naturally from their microeconomic counterparts:

$$M_t = \int \int M_t(z, \zeta_t) \mu_t(a, z) da \; dz \tag{10}$$

4.3. Monetary Block

The central bank directly sets nominal interest rates in the economy according to

$$i_t = r^* + \phi_\pi \pi_t \tag{11}$$

where r^* is the interest rate that would prevail in equilibrium in the absence of any aggregate shocks. The active fiscal model can be solved so long as the interest rate rule is "passive," such that $\phi_\pi < 1$. As such, I set $\phi_\pi = 0$ to examine the properties of the model economy under an interest rate peg.

4.4. Firms and Price Setting

Labor is the only production input in the model economy, such that

$$Y_t = L_t, \tag{12}$$

where Y_t is aggregate real output and L_t is the aggregate number of effective hours worked. Final goods firms are perfectly competitive and face no friction in how they set prices to maximize profits, making wage inflation equal to the final consumption goods' inflation.

Output and employment are demand-determined due to nominal rigidities in the labor market, which are in the style of the decentralized labor union environment of Auclert et al. (2024), which is in turrn related to Hagedorn et al. (2019) (an earlier adopter of sticky wages in a HANK setting) and a modification of Schmitt-Grohé and Uribe (2005). A continuum of decentralized unions hires labor from households and resells it to firms, who differentiate the unions with a constant elasticity of substitution ε_L. Labor supply is demand-determined so that all households work the same number of hours ($h_t(a, z) = L_t/Z$ where $Z = \int \int z\mu(a, z) da \; dz$), and unions are subject to Rotemberg (1982) nominal wage pricing frictions. The result is a nominal forward-looking wage Phillips Curve, which is also the overall Phillips Curve in the economy:

$$\frac{\mathbb{E}_t[d\pi_t]}{dt} = r_t\pi_t - \frac{\varepsilon_\ell}{\theta_w}\frac{L_t}{Z}\int\int\left(h_t(a,z)^{\frac{1}{\eta}} - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)zw_tc_t(a,z)^{-\gamma}\right)da\ dz \tag{13}$$

Inflation today is related to both expected inflation and the average cross-sectional wedge between the disutility of labor and the utility of working for wages, marked down because the unions internalize the effect of supplying more labor on their wage rate.

4.5. Policy Shocks

I assume that aggregate shocks mean-revert at constant rates. As such, they can be written recursively via $d\zeta_{i,t} = -\theta_i\zeta_{i,t}dt$, with the shock of type i at time 0 being given as $\zeta_{i,0}$, or solved forward as a sequence to write

$$\zeta_{i,t} = e^{-\theta_i t}\zeta_{i,0}. \tag{14}$$

Monetary policy shocks revert at a rate of θ_{MP}, while all fiscal shocks revert at a common rate of θ_{Tax}.

4.6. Market Clearing

Aggregate consumption

$$C_t = \int\int c_t(a,z)\mu_t(a,z)da\ dz \tag{15}$$

is equal to aggregate output:

$$Y_t = C_t. \tag{16}$$

Total hours worked are uniform across households:

$$h_t(a,z) = L_t/Z. \tag{17}$$

The asset market clears when net private wealth equal to aggregate government debt:

$$\int\int a\mu_t(a,z)da\ dz = B_t. \tag{18}$$

4.7. HANK Equilibrium

If the amount of initial nominal bonds in the non-stochastic steady state is set exogenously by the government prior to the realization of any shocks, then the initial price level is determined so as to clear the real asset market, much as in Hagedorn (2016). Thereafter, agents can recover the price level using the inflation realized after the initial time period – such that the price level itself need not be tracked as a state variable in the perturbation solution.

An equilibrium given a sequence of aggregate shocks $(\zeta_t)_{t\geq 0}$, an initial wealth and income distribution $\mu_0(a,z)$, and an initial debt level B_0 is therefore a collection of sequences of macroeconomic aggregates

$$(C_t, L_t, Y_t, B_t)_{t\geq 0}$$

and household-level variables and prices

$$(c_t(a,z), h_t(a,z), M_t(z,\zeta_t), \mu_t(a,z), w_t, r_t, i_t, \pi_t)_{t\geq 0}$$

where

i. saver consumption choices $(c_t(a,z))_{t\geq 0}$ solve (5) given prices and aggregates

ii. the distribution of households $\mu_t(a,z)$ evolves according to (6)

iii. labor allocations $(h_{1,t})$ are consistent with the union rule (17)

iv. inflation π_t is consistent with the unions' maximization problem and resulting wage Phillips Curve (13)

such that

1. Macro aggregates $(Y_t, C_t)_{t\geq 0}$ are consistent with production (12) and aggregation (15)

2. real wages w_t are constant and real rates r_t obey the Fisher equation $r_t = i_t - \pi_t$

3. nominal interest rates $(i_t)_{t\geq 0}$ follow the central bank's policy rule (11)

4. Government taxes and transfers across the population and over time $(M_t(z,\zeta_t))_{t\geq 0}$ follow the rule (9) and aggregate to M_t and T_t via (10) and (8)

5. Government debt B_t given taxes T_t and real rates r_t evolves according to (7)

6. The asset market clears, as in (18). By Walras' law, this also implies goods market clearing (16).

5. Calibration

I calibrate my model largely with parameters that are standard in the HANK literature; they are displayed in Table 1. As in McKay et al. (2016), I calibrate the continuous time income process parameters (θ_z, σ_z^2) via simulated method of moments to match the Floden and Lindé (2001) estimates of the permanent component of annual wage autocorrelation and autoregression variance, residualized for age, occupation, education, and other covariates. I similarly calibrate the time discounting parameter ρ to match a real interest rate of 0.5% quarterly, or roughly 2% annually. Real government debt outstanding is set to 67% of annual GDP in the steady state, so that households' average contemporaneous annualized MPC out of a transfer roughly matches those reported in Auclert et al. (2024). I solve for the model's non-stochastic steady state using the methods outlined in Achdou et al. (2021); select moments from this distribution are reported in Table 2.

The slope of the Phillips Curve is also reported in terms of the coefficient describing the passthrough from marginal labor disutility to prices $\frac{\varepsilon_L}{\theta_\pi} h_t v'(h_t)$, where v is the households' labor disutility, to be comparable with the parameters used in Auclert et al. (2024). In Appendix D.2, I simulate the model with different slopes of the Phillips Curve to evaluate the robustness of my findings to this key parameter. Increasing nominal rigidities predictably amplifies the real effects of active fiscal expansion and smooths the transition of prices, while decreasing nominal rigidities does the opposite. Even so, changing the degree of nominal rigidity in the economy leaves the long-term price level dynamics essentially unchanged, nor does it significantly alter the ordering of sacrifice ratios among the different transfer policies.

The marginal distributions of households along assets and incomes are displayed in Figure 1. Since the distribution of assets contains an atom at the borrowing constraint, I display the

Table 1: General HANK Model Parameters

Parameter	Symbol	Value	Source or Target
Households			
Internally Calibrated:			
Quarterly Time Discounting	ρ	0.021	$r = 2\%$ Annually
Idiosyncratic Income Shock Variance	σ_z^2	0.017	Floden and Lindé (2001)
Idiosyncratic Shock Mean Reversion	θ_z	0.034	Floden and Lindé (2001)
Assumed from Literature:			
Relative Risk Aversion	γ	2.0	McKay et al (2016)
Frisch Elasticity of Labor	η	0.5	Chetty (2012)
Labor Market			
Labor Elasticity of Substitution	ε_L	10	Philips Curve slope of 0.07
Rotemberg wage adjustment cost	θ_w	100	Philips Curve slope of 0.07
Government			
steady state government debt	B_{NSS}	2.63	HANK $iMPC_0 \approx 0.40$
Geometric maturity structure of debt	ω	0.043	Avg. maturity of 70 months
Income Tax Rate	τ	0.25	
Taylor Rule Coefficient	ϕ_π	0	Passive monetary policy (peg)
Fiscal Debt Coefficient	κ	0	Active fiscal policy
Shocks			
Mean reversion of fiscal shocks	θ_{Tax}	1.0	

Table 2: HANK Non-Stochastic steady state Statistics

Description	Symbol	Value
Contemporaneous iMPC (Annual)		0.43
Debt to Annual Income	$B_{NSS}/(4Y_{NSS})$	0.67
Correlation btw. Income and Assets	$\text{Corr}(a, z)$	0.56
Share of households with $a = 0$	$\int \mu_{NSS}(0, z)dz$	0.27
Asset Gini Coefficient		0.75
Income Gini Coefficient		0.31

cumulative stationary distribution of assets, followed by the probability density of household incomes. The third plot in Figure 1 depicts the aggregate intertemporal MPCs ("iMPCs") of households in the non-stochastic steady state in response to a year-long transfer that integrates to 1. The iMPCs are aggregated to the annual level to make them comparable with Figures 1 and 2 of Auclert et al. (2024). Households in my model spend roughly 43% of the value of their initial transfer income in the first year when they receive it, 12% a year later, 9% two years later, 7% a year after that, and so on. These iMPCs are roughly consistent with the lower bound presented in Auclert et al. (2024), which uses data from the Italian Survey of Income and Wealth. The plot's dashed lines indicate households' aggregate

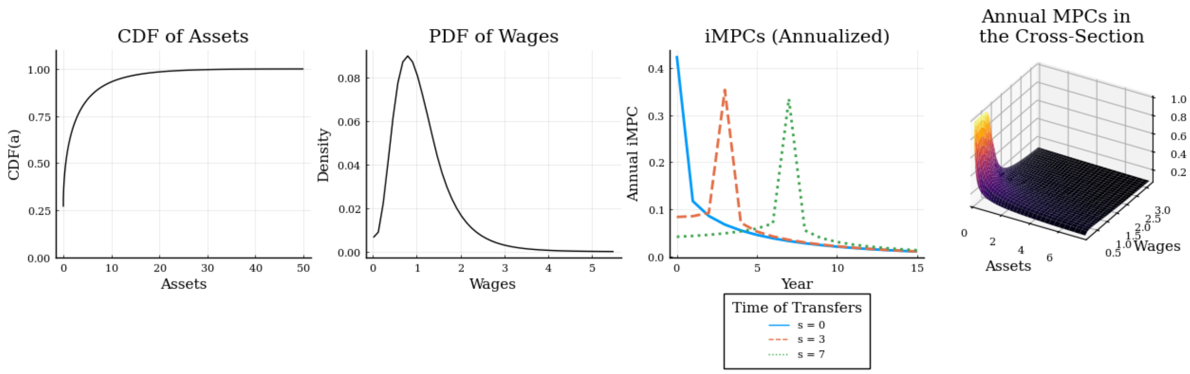

Figure 1: Marginal distributions and marginal propensities to consume in the non-stochastic steady state (both intertemporally and in the cross-section). Assets refer to agents' liquid wealth position a, while wages refers to agents' position in the skill distribution z.

propensity to spend when a transfer is announced 3 and 7 years in advance; the tent-shaped spending patterns are again reminiscent of Auclert et al. (2024).

The final plot in Figure 1 depicts the cross-sectional distribution of households' marginal propensities to consume over 4 quarters out of a change to their liquid wealth, calculated using the Feynman-Kac approach employed in Kaplan and Violante (2018). The average roughly matches the first instantaneous iMPC to a contemporaneous shock reported in the previous graph. As one might expect, most agents with no liquid assets and low income have an MPC of close to 1. This MPC rapidly declines as household wealth increases, or (once wage income becomes high enough) as wage income increases.

I assume fiscal shocks mean revert quickly, with $\theta_{\text{Tax}} = 1.0$. This is intended to better replicate the speed with which stimulus checks may be sent out; after 4 quarters, the fiscal shocks almost entirely dissipate. Since the path of the shock in the absence of further perturbations may be described with equation (14), this also means that the cumulative effect of an initial shock of $\zeta_0^{\text{Tax}} = 0.01$ has the interpretation of a 1%-of-annual-GDP disbursal of lump-sum stimulus checks.[2]

I make fiscal policy active by setting $\kappa = 0$ and monetary policy passive by setting $\phi_\pi = 0$.

[2] For example, if the United States economy in 2019 were to be taken to be the non-stochastic steady state, this would be a spending program of \$210 billion. The 2021 American Rescue Plan's direct stimulus payments amounted to roughly double this amount.

6. HANK Results

6.1. Defining Price Level Sacrifice Ratios

In the numerical experiments wherein I send stimulus checks to high-income and low-income households, I quantitatively show that the cumulation of output gaps is much more sensitive than the change in the price level to household heterogeneity. To make these notions precise, I construct $\mathcal{C}Y_t$, the accumulated increase in GDP relative to the non-stochastic steady state, as

$$\mathcal{C}Y_t \equiv \frac{1}{Y_{NSS}} \int_0^t (Y_s - Y_{NSS}) ds. \tag{19}$$

Cumulative inflation $\mathcal{C}\pi_t$, the total increase in the price level following the shock, can be found by solving the differential equation $\frac{dp_t}{dt} = \pi_t p_t$ forward in time with the initial price level as given:

$$1 + \mathcal{C}\pi_t = \exp\left(\int_0^t \pi_s ds\right). \tag{20}$$

I define the cumulative *price level* sacrifice ratio, the accumulated trade-off as of time t between *annual* real GDP and the change in the price level in response to a shock, as

$$SR_t \equiv (\mathcal{C}Y_t/4)/\mathcal{C}\pi_t$$

Note that this ratio's denominator is slightly different from other definitions of the "sacrifice ratio"; it is analogous to the inverse of the Phillips multiplier statistic introduced by Barnichon and Mesters (2021) (but using the output gap directly instead of the fall in unemployment). Historically, papers like Ball (1994b) measured sacrifice ratios as the cumulative real annualized output gaps correlated with a 1% point decline in inflation over a certain interval of time (perhaps a particular historical period over which shocks to aggregate supply contributed relatively little to the variation of macroeconomic aggregates). By construction, this was a backward-looking quantity; one measured inflation at the end of the interval and subtracted out inflation at the start of the interval, and then divided the sum of output gaps by the resulting quantity.

As discussed in Cochrane (2024), this empirical framework is difficult to map to the modern macroeconomic models like the one discussed in this paper. In both my HANK and

TANK, inflation is zero the moment before the shock, jumps upon impact, and then declines back to zero afterward – leaving the pre-post difference zero asymptotically and rendering the denominator of a conventional sacrifice ratio indeterminate. Moreover, positive output gaps are associated with *declining* inflation after the shock, a conceptual disconnect flagged in Ball (1994a). As such, if one measures initial inflation at the moment of the shock's impact instead of right before, the change in inflation is actually opposite the change in the price level and produces a misleadingly negative "sacrifice ratio."

This is because the standard New Keynesian Phillips Curve is entirely forward-looking; it integrates such that inflation and future output gaps are related by $\int_t^\infty e^{-\rho(t-s)}\widehat{Y}_s ds/\pi_t = 1/\nu$ (the inverse of the slope of the Phillips Curve). To reduce inflation by 1%, the present value of future real GDP must decline by $1/\nu$, an ostensibly constant sacrifice ratio. However, this is a very different quantity than the one measured by Ball (1994b); it pertains to the jump in inflation at the *start* of the interval looking forward, not the end looking back.

As such, I proceed with my cumulative price level sacrifice ratio to measure the total inflation-output trade-off. By replacing the change in the inflation rate in the denominator with the overall change in the price level, I can again meaningfully compare inflation and output over time. This statistic could also be empirically estimated – I provide a rough calculation for the post-COVID period – but the reader should be aware that the sacrifice ratio simulated in the model is slightly conceptually different from what past empirical work has studied.

6.2. Policies that generate the same nominal debt

Suppose the government in the HANK economy decides – reasoning through the equilibrium dynamics of their actions – to send a sequence of transfers to high or low-income households that result in the same long-term nominal debt growth as a 1% of GDP sequence of transfers to all agents in the economy. In the numerical simulation, this would amount to a 1.05% of GDP sequence of transfers to those below median income, or a 0.955% of GDP sequence of transfers to those above median income.

The cumulative output gaps resulting from such a nominal debt-invariant policy are depicted in Figure 2. The red lines depicting the cumulative rise in the price level settle to

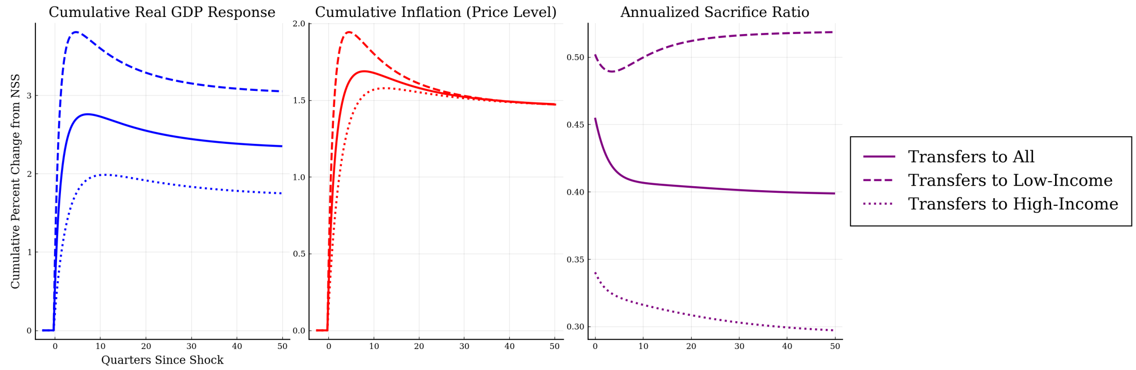

Figure 2: Cumulative impulse response functions in a HANK model, but with real transfers scaled so that the transfers schemes generate the same long-term nominal debt as a uniform transfer to all agents in the economy.

exactly the same level in the second panel; the government designing policy shocks to reach a certain nominal debt is equivalent to the government choosing a long-term price level. Although the total cumulative inflation in all three scenarios is the same, MPC heterogeneity leads transfers targeted to poorer households to generate larger cumulative real output gaps, while transfers directed to richer households generate a smaller cumulative expansion.

In light of this, the price level sacrifice ratios entailed by the different policies are all very different. Transfers sent to poor households raise cumulative annualized GDP by 0.52 percentage points for every 1% increase in the price level, but transfers to the low income only raise cumulative output by 0.30 percentage points for the same long-term inflation.

6.3. Policies that send out the same exogenous real transfers

While governments do often try to assess the long-term debt burden of policies, forecasting the endogenous evolution of nominal bonds in the economy is challenging in the real world. Policies are more often described in terms of their direct fiscal consequences. What, then, is the impact of sending active fiscal transfers amounting to 1% of annual GDP to the low income, the high income, or uniformly across the economy?

I display the cumulative output gaps and increase in the price level for this experiment in Figure 3; the unaccumulated impulse response functions are depicted in Figure 4. The amount of long-term nominal debt generated by the different policies is different in this experiment, and so the price levels do settle to slightly different values in the long term, as depicted in the second panel of Figure 3. However, transfers with the same real value in an

	Transfers to **All**		Transfers to **Low-Income**		Transfers to **High-Income**	
	1 yr	50 qtrs	1 yr	50 qtrs	1 yr	50 qtrs
$\mathcal{C}Y_t/4$	0.66%	0.59%	0.90%	0.73%	0.43%	0.46%
$\mathcal{C}\pi_t$	1.58%	1.47%	1.85%	1.40%	1.34%	1.54%
Sac. Ratio	0.42	0.40	0.49	0.52	0.32	0.30

Table 3: Cumulative annualized output gaps ($\mathcal{C}Y_t/4$), inflation ($\mathcal{C}\pi_t$), and sacrifice ratios for fiscal transfers (amounting to 1% of annual GDP) to different groups in the active fiscal/passive monetary HANK model.

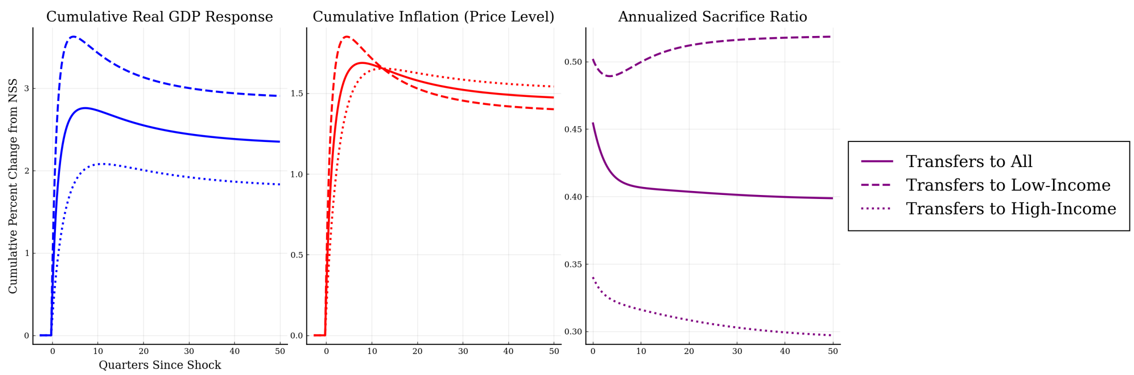

Figure 3: Cumulative impulse response functions in a HANK model. Shocks include 1% of annual GDP increases in transfers to all agents, below-median income agents, and above-median income agents, respectively.

active fiscal, passive monetary framework still lead to nearly the same amount of nominal debt to be inflated away, even if they weren't explicitly designed to do so a priori and even in the presence of automatic stabilizers like income taxes. *Quantitatively*, the amount of cumulative inflation generated by the different policies is therefore still very similar[3] regardless of to whom the transfers were sent, while the paths of cumulative gains to output are very different.

I report the accumulated quantities $\mathcal{C}Y_t/4$, $\mathcal{C}\pi_t$, and the price level sacrifice ratio at different time horizons for the active fiscal/passive monetary setting in Table 3. The first row details the cumulative sum of the output gaps as a percent of annual steady state GDP, respectively accumulated up to 1 year and up to 50 quarters, for transfers to all, below-median income, and above-median income households. Since the transfers are almost entirely paid out after four quarters and accumulate to 1% of annual GDP, this row could

[3]Because the model is linear, the inflation lines in 3 are essentially just a -5% and +4.5% rescaling of the low-income and high-income lines of 2, respectively, as they represent 1% of GDP transfers instead of 1.05% and 0.955% transfers.

also be read as the fiscal transfer multiplier of the different policy shocks. The total rise in the price level for the different transfers and time horizons is reported in the next line. Finally, I report the cumulative sacrifice ratio (the ratio of the first and second lines) in the last row.

Examining the quantitative implications of the policies, transfers to below-median income households boost cumulative annualized output gaps by more than twice as much as transfers to above-median income households and 33% more than untargeted transfers in the first year. After 50 quarters, the amount declines slightly as output overshoots – but transfers to low income households still generate a 59% and 24% larger accumulation of real output gaps than high income transfers and untargeted transfers, respectively. This is despite the fact that the 50-quarter rise in the price level is nearly the same for both untargeted transfers and transfers to low-income households, and actually 9% lower than the rise in the price level associated with transfers to the high income (although in the first year, targeted transfers to the low income do yield significantly more inflation).

In the short run, transfers to low-income agents generate not only a sharper spike in real output following the shock, but also a sharper spike in inflation, as shown in the first two rows of Figure 4. However, the ratio of output to inflation is still higher even in the short term for transfers to the low income as compared to transfers for the high income or untargeted transfers. Over a longer period of time, the inflation response is also less persistent when the transfers are sent to high-MPC agents, in keeping with the intuition developed in earlier sections.

What drives the expansion of real output in the active-fiscal/passive-monetary HANK model after transfer payments go out? Perhaps unsurprisingly, the majority of the response is driven by the increase in households' aggregate demand following an increase in their transfer income net of taxes – particularly because the persistence of the transfers is low. In Figure 5, I decompose the output impulse response function into a component associated with the households' response to transfers themselves (in yellow), the path of real interest rates r (in red) and changes in aggregate demand for labor L (in blue). The paths of each of these inputs, determined in equilibrium, are taken as given by households; the colored regions of the plot depict how each contributes to the total movement of real GDP, which is depicted

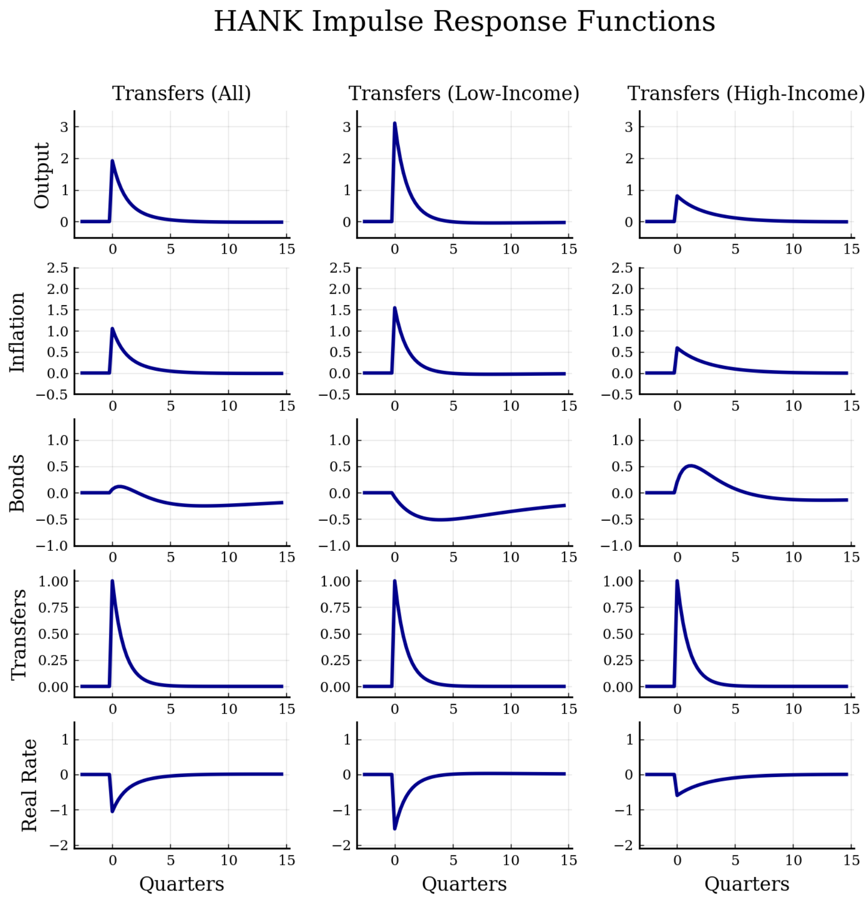

Figure 4: Impulse response functions (unaccumulated) to policy shocks in the HANK environment. All variables are presented as deviations from their quarterly values in the non-stochastic steady state except for transfers, which are reported as a percentage of annual real GDP in the non-stochastic steady state.

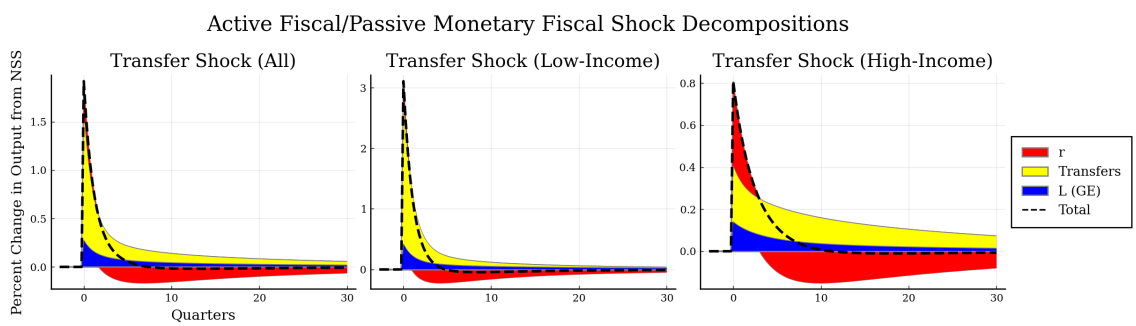

Figure 5: Decompositions of the real output impulse response function in an active fiscal/passive monetary HANK. Each channel represents the heterogeneous agents' response to i) real interest rates and bond prices (in red), ii) transfers (in yellow), and general equilibrium changes in labor demand (in blue). The colored regions add up to the dashed black line.

in the black dashed line. While an increase in employment and labor income does contribute to the expansion (and the increase in aggregate equilibrium labor is what produces the goods that households consume), the rise in aggregate consumption is predominantly driven by the increase in net transfer income that high-MPC households receive (the policy's direct effect). When low-MPC households receive the checks, general equilibrium effects play a larger role in the smaller real GDP response; households are motivated to spend following the decline in real rates following inflation, and then to save again to rebuild their precautionary savings following the boom once real rates of return have recovered.

How do these price-level sacrifice ratios compare to the data following the COVID-19 pandemic? If trend inflation was still 2% in the years following the pandemic, then the Consumer Price Index rose by nearly 9.5% in excess of that trend from 2021Q2 to 2024Q2. The sum of the quarterly output gaps estimated by the Congressional Budget Office during that period accumulates to 4.4%, for a cumulative ratio of 0.47. As can be inferred from Table 3, this real-world output/inflation ratio is roughly in line with a model scenario where transfers are issued slightly disproportionately to low-income households. This is broadly consistent with the fact that some of the actual federal stimulus in the period came from expansions to child tax credits, unemployment insurance, and other existing welfare benefits. Of course, other shocks besides changes to fiscal policy affected the U.S. during this time period – and the model significantly over-predicts the amount of inflation conditional on a $1.9 trillion stimulus package like the 2021 American Rescue Plan in the absence of other forces affecting the economy.[4] Even so, the model's simulated macroeconomic trade-off appears to be close to what occurred in the early 2020s.[5]

One could also calculate the cumulative effects in Figure 3 using the present value of inflation and the present value of output gaps, instead of the definitions used in equations (19) and (20). In the appendix, I show that this only slightly changes the picture; the dispersion in the red price level lines shrinks by a factor of only one quarter. This indicates that most of the small spread in the eventual price level (and eventual nominal bonds) in Figure 3 is due to the presence of automatic stabilizers induced by income taxation, as opposed to timing effects that follow when debt is inflated away earlier and faster.

[4]An untargeted stimulus package of slightly less than 8% of GDP results in an inflation multiplier of roughly 1.65 after 12 quarters, exceeding the entire 10% rise in the price level observed in the data.

[5]I demonstrate the effect of changing the slope of the Phillips Curve in Appendix D.2 on the cumulative impulse response functions and the price level sacrifice ratios.

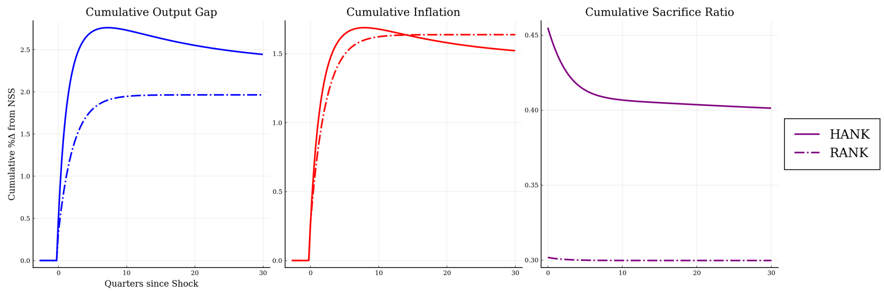

Figure 6: Cumulative GDP, inflation, and annualized price level sacrifice ratios in the HANK economy to a similarly-calibrated RANK economy, when fiscal transfers are sent out to all agents.

6.4. Comparing HANK to RANK

As discussed in Section 3, incomplete markets and borrowing constraints do little to change the accounting by which the equilibrium inflation must adjust the value of real aggregate liquid assets back to their steady-state levels. In the absence of automatic stabilizers that raise tax revenue from output gaps, the cumulative present value of inflation will be exactly the same in complete and incomplete markets, so long as the initial debt and the present value of the path of surpluses in both economies are the same. Incomplete markets raise the average MPC in the economy, however, so the output response to an active fiscal response in incomplete markets will be larger – raising cumulative sacrifice ratios in HANK as opposed to RANK.

More realistic additions to the model, like income taxes, and potential differences in the timing of inflation slightly complicate this picture but leave the overall message unchanged. In Figure 6, I repeat the experiment of sending fast-reverting fiscal transfers valued at 1% of GDP to all agents in the economy, but now in both the HANK setting and in a complete markets RANK environment. The RANK model is calibrated to have the same steady-state interest rate, debt-to-GDP ratio, and income tax profile as in the HANK setting.

In the first panel, the high MPCs of constrained households generated by HANK raise the stimulative effect of the transfers on real output compared to RANK. Due to the Phillips' curve, inflation arrives slightly more swiftly in the calibrated HANK, while income taxes also draw down some of the new nominal private assets in the economy. Both of these forces make cumulative inflation in HANK actually slightly *lower* in the long term, compared to RANK – but the effect is again relatively small, and the price level rises by nearly the same amount in both economies. This too makes the cumulative sacrifice ratios after fifty quarters much higher in HANK (0.40) compared to RANK (0.30). Naturally, the comparison is even more stark if the transfers are targeted to low-income agents – while if transfers are targeted to high income households, the sacrifice ratios in HANK and RANK settle to nearly the same value (again referring to Table 3).

7. Discussion

Because low-income households have low liquid wealth and high marginal propensities to consume, sending deficit-financed transfers to them leads to a sharp boost in output. However, if the central bank does not raise nominal interest rates in response to inflation, then the distribution of transfer recipients has little impact on how much inflation transpires. Under an interest rate peg without a Taylor rule, two exogenous fiscal transfer programs generate roughly the same amount of nominal debt regardless of their targeting. Real debt in the model is stationary: inflation then accumulates until the nominal assets issued by the government and held by households as assets have returned to steady state levels, regardless of who received the funds. As such, cumulative inflation is not sensitive to targeting or heterogeneity in MPCs.

Transfers to the low-income thus generate larger amounts of GDP relative to the amount of inflation they produced, compared to when the checks go to wealthier high-income segments of the population. This is consistent with the baseline Phillips Curve; when output rises quickly, firms take time to adjust their prices and respond to future expected output gaps, not previous ones. This leads the overall rise in the price level to trail a sharp rise in output. Conversely, this dynamic has strong implications for "sacrifice ratios": abating inflation by cutting transfers to the low-income depresses real GDP by much more than similar inflation abatement accomplished by lump-sum tax increases on the rich, as sacrifice ratios themselves are positively related to the speed with which the output gaps occur.

The intuition that the price level might strongly depend on how some households behave more like "savers" or "spenders" after receiving their checks is also not quantitatively supported in a HANK model. As Auclert et al. (2024) notes, optimizing agents will eventually want to spend the present value of whatever they receive, such that the present value of iMPCs aggregates to one, even if they smooth that consumption spending over time. Eventually, for the asset market to clear and for the economy to return to its non-stochastic steady state, inflation occurs to bring nominal private assets back to stable real levels.

When this is the case, one can predict the long-term inflationary impact of a policy without much knowledge of its distributional consequences or implications for employment and output. But is this the case? Less conventional, but perhaps important, theoretical complications could emerge if models contain behavioral agents with MPCs that are truly zero, such as in Auclert et al. (2023b), leading them to act as a permanent real asset sink. Inflation might play a less predictable, and perhaps reduced, role in the equilibrium dynamics of such models. Empirically, there also appears to be an opening for more work examining how inflation does or does not ensue when governments do not have a credible plan to pay down their debt through conventional means following unexpected deficit spending. Ultimately, recent theories of the price level and models with meaningful heterogeneity present new ways to understand how fiscal and monetary policy interact to influence macroeconomic aggregates – potentially with strong implications for policy in the real world.

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Appendix A. Simple Model Derivations

Appendix A.1. 5 Equations for a Simple TANK Model

In this section, I sketch a simple TANK model to intuit how transfers to high MPC agents might yield more output but very similar levels of inflation compared to transfers to low MPC households when fiscal policy is active and monetary policy is passive. The simple model includes two households, a government that can send exogenous transfers to each by running deficits and borrowing, and a basic New Keynesian Phillips Curve. For simplicity, I here consider a model that establishes determinacy using the FTPL, while my HANK model's determinacy comes from the DTPL. However, I parameterize and simulate a slightly more complicated TANK model in **??** that can display either a DTPL equilibrium or an FTPL equilibrium and show that the results for both are qualitatively similar.

The first representative household is a measure $1-\mu$ continuum of forward-looking agents who collectively hold the stock government debt (a "saver" household, labeled 1), and is of measure $1-\mu$. These households receive transfers $M_{1,t}$ from the government, which are equal to zero in the non-stochastic steady state, and additionally pay the government's steady state interest expense T_{NSS}. They choose consumption $c_{1,t}$ in accordance with an Euler equation, derived in Appendix A, where ρ is the rate of time discounting, γ^{-1} is the elasticity of intertemporal substitution, and r_t is the real interest rate:

$$\frac{\mathbb{E}_t[dc_{1,t}]}{dt}\frac{1}{c_{1,t}} = \gamma^{-1}\left[r_t - \rho\right]. \tag{A.1}$$

The second, of mass μ, is a continuum of households who are constrained to consume their income every period. These agents ("spenders," labeled 2), set their consumption $c_{2,t}$ equal to labor income from working plus income from transfers they receive from the government.

$$c_{2,t} = Y_t + M_{2,t} \tag{A.2}$$

where $Y_t = L_t$ is aggregate output and hours worked, aggregate demand is

$$Y_t = (1-\mu)c_{1t} + \mu c_{2t} \tag{A.3}$$

and the real wage rate is equal to 1 – as would be the case in a model where wages are nominally rigid and the output price sector is perfectly competitive, making the real wage perfectly acyclic.

The government issues nominal bonds of real value B_t at a real interest rate of r_t to pay for deficits. Tax revenue is $T_t = T_{NSS} - \frac{1}{1-\mu}M_{1,t} - \frac{1}{\mu}M_{2,t}$, where $T_{NSS} = r_{NSS}B_{NSS}$. As such, the real stock of government debt evolves according to

$$\frac{dB_t}{dt} = -T_t + r_t B_t \tag{A.4}$$

and the central bank fixes the nominal interest rate, such that $i_t = i$. With the Fisher equation, this means that $r_t = i - \pi_t$, where π_t is the rate of inflation.

The New Keynesian Phillips Curve is then

$$\rho\pi_t = \frac{\mathbb{E}_t[d\pi]}{dt} + \nu\widehat{Y}_t \tag{A.5}$$

where $\widehat{Y}_t$ is the percent deviation of real GDP from its value in the steady state (the output gap) ν is the slope of the Phillips Curve.

Suppose now that the economy is in steady state (with zero inflation and no transfers besides the lump-sum ones used to balance the budget) at time t when the government announces that it intends to send transfers to one household but not the other by temporarily raising either $M_{1,t}$ or $M_{2,t}$ from their steady state values by running deficits that will never be repaid with future taxes. After a short period of time, these deficits return to zero. To analyze such an experiment, I consider the equations one-by-one.

Appendix A.2. The Phillips Curve and Cumulative Inflation and Output Gaps

Equation (A.5) can be integrated forward to write

$$\pi_t = \nu\int_t^{\infty} e^{-\rho s}\widehat{Y}_s ds$$

Accumulating inflation from time 0 to a terminal time T, I define $\mathcal{C}\pi_T$ as the rise in the price level by time T and approximate it as

$$\mathcal{C}\pi_T \equiv \exp\left(\int_0^T \pi_t dt\right) - 1 \approx \int_0^T \pi_t dt = \nu \int_0^T \left(\int_t^\infty e^{-\rho s}\widehat{Y}_s ds\right) dt$$

The timing of the output gaps matter. The region being integrated over is the triangle defined by $0 \leq t \leq T$ and $t \leq s \leq \infty$. This is the same region as the one bounded by $0 \leq s \leq T$ and $0 \leq t \leq \min(s, T)$. Changing the order of integration,

$$= \nu \int_0^\infty \int_0^{\min(s,T)} e^{-\rho s}\widehat{Y}_s dt\ ds = \nu \int_0^T se^{-\rho s}\widehat{Y}_s ds + \nu \int_T^\infty Te^{-\rho s}\widehat{Y}_s ds$$

and taking $T \to \infty$,

$$\int_0^\infty \pi_t dt = \nu \int_0^\infty te^{-\rho t}\widehat{Y}_t dt \tag{A.6}$$

Suppose the output gaps jump and decay back to steady state at a rate of λ_Y, such that $\widehat{Y}_t = \lambda e^{-\lambda t}\mathcal{C}Y_\infty$, where $\mathcal{C}Y_\infty \equiv \int_0^\infty \widehat{Y}_t dt$ is the cumulative output gap over time. In that case,

$$\int_0^\infty \pi_t dt = \nu \int_0^\infty te^{-\rho t}\lambda e^{-\lambda t}\mathcal{C}Y_\infty dt = \nu\lambda\mathcal{C}Y_\infty \int_0^\infty te^{-(\rho+\lambda)t}dt$$

such that

$$\mathcal{C}\pi_\infty / \mathcal{C}Y_\infty = \nu\frac{\lambda}{(\lambda+\rho)^2}$$

If $\rho \approx 0$, then $\mathcal{C}\pi_\infty/\mathcal{C}Y_\infty \approx \nu/\lambda$. The asymptotic amount of cumulative inflation relative to cumulative output tends to increase with the slope of the Phillips Curve, but *decrease* when output rises faster. More output in a given time increment increases the amount of inflation, but nominal rigidities imply that faster growth in output mean that prices cannot, in a sense keep up. The Phillips Curve is forward looking; previous output gaps are already sunk from the perspective of the firm. If a lot of growth happens quickly and then subsides, that past growth no longer matters for period t inflation; all that matters are future output gaps.

With similar logic, the present value of inflation discounted by ρ is

$$\int_0^\infty e^{-\rho t}\pi_t dt = \frac{\nu}{\rho}\int_0^\infty (e^{-\rho t} - e^{-2\rho t})\widehat{Y}_t dt$$

Appendix A.3. Debt Evolution: Inflation and Nominal Debt

Begin with the debt evolution equation:

$$\frac{dB_t}{dt} = -T_t + r_t B_t$$

Solving the ODE forward with an integrating factor of $\int_t^\tau r_s ds$ and assuming the real value of debt does not explode,

$$B_t = \int_t^\infty e^{-\int_t^\tau r_s ds} T_\tau d\tau$$

Dividing by steady state real debt, taxes, and real interest rates as (B, T, r) (no time indexes) and writing $B_t = Be^{\widehat{B}_t}$, $T_t = Te^{\widehat{T}_t}$, $r_t = \widehat{r}_t + r$,

$$e^{\widehat{B}_t} = \int_t^\infty e^{-\int_t^\tau (\widehat{r}_s + r) ds} \frac{T}{B} e^{\widehat{T}_\tau} d\tau$$

Log-linearizing, including writing $\exp\left(-\int_t^\tau \widehat{r}_s ds\right) \approx 1 - \int_t^\tau \widehat{r}_s ds$

$$(1+\widehat{B}_t) \approx \frac{T}{B}\int_t^\infty e^{-(\tau-t)r}\left(1 - \int_t^\tau \widehat{r}_s ds\right)(1+\widehat{T}_\tau)d\tau \approx \frac{T}{B}\int_t^\infty e^{-(\tau-t)r}\left(1 - \int_t^\tau \widehat{r}_s ds + \widehat{T}_\tau\right)d\tau$$

where the second approximation follows from the cross-terms of the hatted variables being very small. Note that with a $u = -(\tau - t)r$ substitution, $\frac{T}{B}\int_t^\infty e^{-(\tau-t)r} d\tau = \frac{1}{r}\frac{T}{B}\int_0^{-\infty} e^u du = 1$. As such,

$$\widehat{B}_t \approx \frac{T}{B}\int_t^\infty e^{-(\tau-t)r}\left(\widehat{T}_\tau - \int_t^\tau \widehat{r}_s ds\right)d\tau$$

Note that as $r_t = i_t - \pi_t$, it follows that $\widehat{r}_t = \widehat{i}_t - \widehat{\pi}_t$, where the hatted variables denote deviations from the non-stochastic steady state.

$$\widehat{B}_t \approx \frac{T}{B}\int_t^\infty e^{-(\tau-t)r}\left(\widehat{T}_\tau - \int_t^\tau (\widehat{i}_s - \widehat{\pi}_s)ds\right)d\tau$$

If the central bank sets interest rates according to a Taylor rule like $\widehat{i}_t = \phi_\pi \widehat{\pi}_t$ but $\phi_\pi < 1$, then

$$\widehat{B}_t \approx \frac{T}{B}\int_t^\infty e^{-(\tau-t)r}\left(\widehat{T}_\tau + (1-\phi_\pi)\int_t^\tau \widehat{\pi}_s ds\right)d\tau$$

such that

$$(1-\phi_\pi)\int_t^\infty e^{-(\tau-t)r}\left(\int_t^\tau \widehat{\pi}_s ds\right)d\tau = \frac{1}{r}\widehat{B}_t - \int_t^\infty e^{-(\tau-t)r}\widehat{T}_\tau d\tau.$$

The region demarcated by $t \le s \le \tau$ and $t \le \tau \le \infty$ can be equivalently demarcated by $s \le \tau \le \infty$ and $t \le s \le \infty$. As such,

$$\int_t^\infty e^{-(\tau-t)r}\left(\int_t^\tau \widehat{\pi}_s ds\right)d\tau = \int_t^\infty \int_s^\infty e^{-(\tau-t)r}\widehat{\pi}_s d\tau\ ds = \int_t^\infty \widehat{\pi}_s \left(\int_s^\infty e^{-(\tau-t)r} d\tau\right) ds$$

$$= \frac{1}{r}\int_t^\infty e^{-(s-t)r}\widehat{\pi}_s ds$$

Using the fact that $r = \frac{T}{B}$:

$$(1-\phi_\pi)\int_t^\infty e^{-(\tau-t)r}\widehat{\pi}_\tau d\tau = \widehat{B}_t - \frac{T}{B}\int_t^\infty e^{-(\tau-t)r}\widehat{T}_\tau d\tau. \tag{A.7}$$

Setting $\phi_\pi = 0$ yields the expression in the main text.

Appendix A.3.1. Example: Exponentially decaying inflation

Suppose $\mathcal{C}\pi_\tau = (1-e^{-\lambda_\pi(\tau-t)})\mathcal{C}\pi_\infty$ for $\tau \ge t$. Note that this implies π_t jumps by a factor of $\lambda\mathcal{C}\pi_\infty$ on impact, and mean reverts with an exponential rate of λ, such that $\pi_\tau = \lambda_\pi e^{-\lambda_\pi(\tau-t)}\mathcal{C}\pi_\infty$. Then the present value of the path of inflation is

$$\int_t^\infty e^{-(\tau-t)r}\widehat{\pi}_\tau d\tau = \int_t^\infty e^{-(\tau-t)(r+\lambda_\pi)}\lambda_\pi \mathcal{C}\pi_\infty d\tau = \frac{\lambda_\pi}{r+\lambda_\pi}\mathcal{C}\pi_\infty$$

such that

$$\int_t^\infty \widehat{\pi}_\tau d\tau = -\left(1+\frac{r}{\lambda}\right)\frac{T}{B}\int_t^\infty e^{-(\tau-t)r}\widehat{T}_\tau d\tau.$$

Note that $r\frac{T}{B} = r^2 \approx 0$ when r is small, so the effect of the timing of the output gaps on cumulative inflation is small if inflation mean reverts with a half life of a few quarters.

Appendix A.4. TANK Euler Equation

I derive the saver household's Euler equation with bonds in the utility function; to obtain the standard Euler equation, I can set $\psi = 0$. For clarity in the following section, I drop the "1" subscripts from the attendant variables, but the quantities in question of course pertain to agent 1 in the simple TANK model. The saver household's problem is

$$\begin{aligned}
&\max_{(c_{1,t})_{t\geq 0}} \mathbb{E} \int_0^\infty e^{-\rho t} \left[\frac{c_{1,t}^{1-\gamma}}{1-\gamma} - \frac{h_{1,t}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \psi \frac{a_t^{1-\gamma_b}}{1-\gamma_b} \right] dt \\
&\text{s.t. } \frac{da_t}{dt} = (1-\tau) w_t h_{1,t} + r_t a_t + M_{1,t} - c_t \\
&\lim_{T\to\infty} \mathbb{E}[e^{-\int_0^T r_t dt} a_T] \geq 0
\end{aligned} \tag{A.8}$$

The Hamilton-Jacobi Bellman equation is (suppressing the value function's dependence on aggregate shocks by subsuming them into the time index)

$$\rho V_t(a) = \max_{c_{1,t}} \left\{ \left[\frac{c_{1,t}^{1-\gamma}}{1-\gamma} - \frac{h_{1,t}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \psi \frac{a_t^{1-\gamma_b}}{1-\gamma_b} \right] + \frac{\partial V_t(a)}{\partial a} [(1-\tau) w_t h_{1,t} + r_t a_t + M_{1,t} - c_t] + \frac{\mathbb{E}_t[\partial V_t(a)]}{\partial t} \right\}$$

Taking first-order conditions,

$$c_{1,t}^{-\gamma} = \frac{\partial V_t(a)}{\partial a}$$

And with the Envelope Theorem,

$$\begin{aligned}
\rho \frac{\partial V_t(a)}{\partial a} =& \psi a_t^{-\gamma_b} + \frac{\partial}{\partial a} \left(\frac{\partial V_t(a)}{\partial a} [(1-\tau) w_t h_{1,t} + r_t a_t + M_{1,t} - c_t] \right) + \frac{\mathbb{E}_t[\partial(\partial V_t(a)/\partial a)]}{\partial t} \\
=& \psi a_t^{-\gamma_b} + \frac{\partial^2 V_t(a)}{\partial a^2} [(1-\tau) w_t h_{1,t} + r_t a_t + M_{1,t} - c_t] + r_t \frac{\partial V_t(a)}{\partial a} + \frac{\mathbb{E}_t[d(\partial V_t(a)/\partial a)]}{\partial t}
\end{aligned}$$

$$\Rightarrow (\rho - r_t) \frac{\partial V_t(a)}{\partial a} = \psi a_t^{-\gamma_b} + \frac{\partial^2 V_t(a)}{\partial a^2} \frac{da}{dt} + \frac{\mathbb{E}_t[d(\partial V_t(a)/\partial a)]}{\partial t}$$

The total time derivative of the expected shadow price of consumption $\frac{\partial V_t(a)}{\partial a}$ is

$$\frac{\mathbb{E}_t[d(\partial V_t(a)/\partial a)]}{dt} = \frac{\partial^2 V_t(a)}{\partial a^2} \frac{da}{dt} + \frac{\mathbb{E}_t[\partial(\partial V_t(a)/\partial a)]}{\partial t}$$

such that the shadow price evolves according to

$$\Rightarrow (\rho - r_t)\frac{\partial V_t(a)}{\partial a} = \psi a_t^{-\gamma_b} + \frac{\mathbb{E}_t[d(\partial V_t(a)/\partial a)]}{dt}$$

Plugging in the first-order condition,

$$\Rightarrow (\rho - r_t)c_{1,t}^{-\gamma} = \psi a_t^{-\gamma_b} + \frac{\mathbb{E}_t[d(c_{1,t}^{-\gamma})]}{dt}$$

where with the chain rule, $\frac{\mathbb{E}_t[d(c_{1,t}^{-\gamma})]}{dt} = -\gamma c_{1,t}^{-\gamma-1}\frac{\mathbb{E}_t[dc_{1,t}]}{dt}$. Rearranging,

$$\frac{\mathbb{E}_t[dc_{1,t}]}{dt}\frac{1}{c_{1,t}} = \gamma^{-1}\left[r_t + \psi c_{1,t}^{\gamma} a_t^{-\gamma_b} - \rho\right].$$

Appendix A.4.1. The saver household's linearized policy function

The saver household's budget constraint states that

$$\frac{da}{dt} = r_t a_t + y_t - c_t$$

where a is the household's asset position, and y_t is their total income (including transfers). Using $e^{-\int_\tau^t r_s ds}$ as an integrating factor,

$$e^{-\int_\tau^t r_s ds}\frac{da}{dt} - e^{-\int_\tau^t r_s ds} r_t a_t = e^{-\int_\tau^t r_s ds}[y_t - c_t]$$

$$\Rightarrow \quad \frac{d}{dt}\left[e^{-\int_\tau^t r_s ds} a_t\right] = e^{-\int_\tau^t r_s ds}[y_t - c_t]$$

Integrating forward to time T,

$$\int_\tau^T \frac{d}{dt}\left[e^{-\int_\tau^t r_s ds} a_t\right] dt = \int_\tau^T e^{-\int_\tau^t r_s ds}[y_t - c_t]dt$$

such that

$$e^{-\int_\tau^T r_s ds} a_T - e^{-\int_\tau^\tau r_s ds} a_\tau = \int_\tau^T e^{-\int_\tau^t r_s ds}[y_t - c_t]dt$$

Thus

$$a_\tau = \int_\tau^T e^{-\int_\tau^t r_s ds}[c_t - y_t]dt + e^{-\int_\tau^T r_s ds} a_T$$

And since the consumer's TVC and no-Ponzi condition stipulates $\lim_{T\to\infty} \mathbb{E}_\tau[e^{-\int_\tau^T r_s ds} a_T] = 0$,

$$a_t = \mathbb{E}_t \left[\int_t^\infty e^{-\int_t^\tau r_s ds}[c_\tau - y_\tau]d\tau \right]$$

where I have interchanged the τ and t indexes, for clarity. Households choose assets to fund the expected present value of their consumption that their expected future income will not cover.

$$\mathbb{E}_t \left[\int_t^\infty e^{-\int_t^\tau r_s ds} c_\tau d\tau \right] = a_t + \mathbb{E}_t \left[\int_t^\infty e^{-\int_t^\tau r_s ds} y_\tau d\tau \right]$$

Log linearizing around the NSS,

$$\mathbb{E}_t \left[\int_t^\infty e^{-(\tau-t)r - \int_t^\tau \widehat{r}_s ds} c e^{\widehat{c}_\tau} d\tau \right] = a e^{\widehat{a}_t} + \mathbb{E}_t \left[\int_t^\infty e^{-r(t-T) - \int_t^\tau \widehat{r}_s ds} y e^{\widehat{y}_\tau} d\tau \right]$$

$$\mathbb{E}_t \left[\int_t^\infty e^{-(\tau-t)r} \left(1 - \int_t^\tau \widehat{r}_s ds\right) c(1+\widehat{c}_\tau) d\tau \right] = a e^{\widehat{a}_t} + \mathbb{E}_t \left[\int_t^\infty e^{-(\tau-t)r} \left(1 - \int_t^\tau \widehat{r}_s ds\right) y(1+\widehat{y}_\tau) d\tau \right]$$

$$\mathbb{E}_t \left[c \int_t^\infty e^{-(\tau-t)r} \left(1 - \int_t^\tau \widehat{r}_s ds + \widehat{c}_\tau\right) d\tau \right] = a(1+\widehat{a}_t) + \mathbb{E}_t \left[y \int_t^\infty e^{-(\tau-t)r} \left(1 - \int_t^\tau \widehat{r}_s ds + \widehat{y}_\tau\right) d\tau \right]$$

$$\mathbb{E}_t \left[c \int_t^\infty e^{-(\tau-t)r} \left(\widehat{c}_\tau - \int_t^\tau \widehat{r}_s ds\right) d\tau \right] = a\widehat{a}_t + \mathbb{E}_t \left[y \int_t^\infty e^{-(\tau-t)r} \left(\widehat{y}_\tau - \int_t^\tau \widehat{r}_s ds\right) d\tau \right]$$

and since at steady state $y = c$,

$$\int_t^\infty e^{-(\tau-t)r} \mathbb{E}_t \left[\widehat{c}_\tau\right] d\tau = \frac{a}{y}\widehat{a}_t + \int_t^\infty e^{-(\tau-t)r} \mathbb{E}_t \left[\widehat{y}_\tau\right] d\tau$$

The Euler equation can then be log-linearized, with the understanding that $r = \rho$:

$$c\frac{\mathbb{E}_t[d\widehat{c}_t]}{dt} = \gamma^{-1}\left[r(1+\widehat{r}_t) - \rho\right] c(1+\widehat{c}_t) \;\Rightarrow\; \frac{\mathbb{E}_t[d\widehat{c}_t]}{dt} = \gamma^{-1}\widehat{r}_t$$

Taking expectations as of time $\tau < t$,

$$\frac{\mathbb{E}_\tau[d\widehat{c}_t]}{dt} = \gamma^{-1}\mathbb{E}_\tau[\widehat{r}_t]$$

such that integrating forward,

$$\underbrace{\int_{\tau}^{T} \frac{\mathbb{E}_{\tau}[d\widehat{c}_t]}{dt}}_{\mathbb{E}_{\tau}[\widehat{c}_T]-c_{\tau}} = \int_{\tau}^{T} \gamma^{-1}\mathbb{E}_{\tau}[\widehat{r}_t]dt$$

and returning to my standard time index notation,

$$\mathbb{E}_{\tau}[\widehat{c}_{\tau}] = c_t + \int_t^{\tau} \gamma^{-1}\mathbb{E}_t[\widehat{r}_s]ds$$

Substituting into the previous intertemporal budget constraint,

$$\int_t^{\infty} e^{-(\tau-t)r}\left(c_t + \int_t^{\tau}\gamma^{-1}\mathbb{E}_t[\widehat{r}_s]ds\right)d\tau = \frac{a}{y}\widehat{a}_t + \int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t\left[\widehat{y}_{\tau}\right]d\tau$$

$$\Rightarrow \int_t^{\infty} e^{-(\tau-t)r}c_t d\tau + \int_t^{\infty} e^{-(\tau-t)r}\left(\int_t^{\tau}\gamma^{-1}\mathbb{E}_t[\widehat{r}_s]ds\right)d\tau = \frac{a}{y}\widehat{a}_t + \int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t\left[\widehat{y}_{\tau}\right]d\tau$$

Changing the order of integration in the second integral and solving the first:

$$\Rightarrow \frac{1}{r}c_t + \gamma^{-1}\int_t^{\infty}\left(\int_s^{\infty} e^{-(\tau-t)r}d\tau\right)\mathbb{E}_t[\widehat{r}_s]ds = \frac{a}{y}\widehat{a}_t + \int_t^{\infty}\mathbb{E}_t\left[\widehat{y}_{\tau}\right]e^{-(\tau-t)r}d\tau$$

$$\Rightarrow \frac{1}{r}c_t + \gamma^{-1}\frac{1}{r}\int_t^{\infty} e^{-(s-t)r}\mathbb{E}_t[\widehat{r}_s]ds = \frac{a}{y}\widehat{a}_t + \int_t^{\infty}\mathbb{E}_t\left[\widehat{y}_{\tau}\right]e^{-(\tau-t)r}d\tau$$

And since in the simple TANK model $r = \rho$ and $\widehat{r}_t = \widehat{i}_t - \widehat{\pi}_t$:

$$c_t = \rho\int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t\left[\widehat{y}_{\tau}\right]d\tau + \rho\frac{a}{y}\widehat{a}_t - \gamma^{-1}\int_t^{\infty} e^{-(\tau-t)r}\mathbb{E}_t[\widehat{i}_{\tau} - \widehat{\pi}_{\tau}]d\tau \tag{A.9}$$

The forward-looking household's linearized MPC out of a NPV income shock of 1 is equal to ρ, if real interest rates are unchanged. This is also the household's MPC out of liquid wealth, where the liquid wealth change is also in terms of a percentage of steady state income. Note that from the perspective of when a shock is realized, $\widehat{a}_t = 0$ if the stock of the household's savings does not jump on impact.

Appendix A.5. Combining Equations

Suppose nominal interest rates are fixed and the path of surpluses is exogenously set for active fiscal policy. Then, the second term is equal to the (negative) present present value of future inflation, which is from the section on the government budget deficit equal to the present discounted value of expected deficits.

$$\widehat{c}_t = \rho \int_t^{\infty} e^{-(\tau-t)r} \mathbb{E}_t \left[\widehat{y}_\tau \right] d\tau + \rho \frac{a}{y} \widehat{a}_t + \gamma^{-1} \left(\widehat{B}_t - \frac{T}{B} \int_t^{\infty} e^{-(\tau-t)r} \mathbb{E}_t [\widehat{T}_\tau] d\tau \right) \tag{A.10}$$

Deficits induce inflation which entail a reduction in real rates, stimulating intertemporal substitution and consumption apart from the change in income.

Appendix A.6. Intertemporal Keynesian Cross (TANK)

If total GDP is

$$Y_t = (1-\mu)c_{1t} + \mu c_{2t} = (1-\mu)y_{1t} + \mu y_{2t} = L_t$$

and $c_{2t} = y_{2t}$, such that $c_{1t} = y_{1t}$, then

$$Y e^{\widehat{Y}_t} = (1-\mu) y_1 \widehat{y}_{1t} + \mu y_2 \widehat{y}_{2t} \quad \Rightarrow \quad \widehat{Y}_t = (1-\mu)\frac{y_1}{Y}\widehat{y}_{1t} + \mu \frac{y_2}{Y}\widehat{y}_{2t}$$

where htm income is

$$y_{2t} = w_t L_t - T_{2t} \;\Rightarrow\; y_2 e^{\widehat{y}_{2t}} = wY e^{\widehat{w}_t + \widehat{Y}_t} - T_2 e^{\widehat{T}_{2t}}$$

$$\Rightarrow \quad y_2 \widehat{y}_{2t} = wY(\widehat{w}_t + \widehat{Y}_t) - T_2 \widehat{T}_{2t}$$

If $w_t = 1$,

$$\Rightarrow \quad \widehat{y}_{2t} = \frac{Y}{y_2}\widehat{Y}_t - \frac{T_2}{y_2}\widehat{T}_{2t}$$

Thus

$$\widehat{Y}_t = (1-\mu)\frac{y_1}{Y}\widehat{y}_{1t} + \mu \frac{y_2}{Y}\left(\frac{Y}{y_2}\widehat{Y}_t - \frac{T_2}{y_2}\widehat{T}_{2t} \right)$$

$$(1-\mu)\widehat{Y}_t = (1-\mu)\frac{y_1}{Y}\widehat{y}_{1t} - \mu\frac{T_2}{Y}\widehat{T}_{2t} \;\Rightarrow\; \widehat{Y}_t = \frac{y_1}{Y}\widehat{y}_{1t} - \frac{\mu}{1-\mu}\frac{T_2}{Y}\widehat{T}_{2t}$$

$$\widehat{y}_{1t} = \frac{Y}{y_1}\widehat{Y}_t + \frac{\mu}{1-\mu}\frac{T_2}{y_1}\widehat{T}_{2t}$$

Substituting this into the sequential policy function,

$$\begin{aligned}\frac{Y}{y_1}\widehat{Y}_t + \frac{\mu}{1-\mu}\frac{T_2}{y_1}\widehat{T}_{2t} = \rho\int_t^\infty e^{-(\tau-t)r}\mathbb{E}_t\left[\frac{Y}{y_1}\widehat{Y}_t + \frac{\mu}{1-\mu}\frac{T_2}{y_1}\widehat{T}_{2t}\right]d\tau + \\ \rho\frac{a}{y_1}\widehat{a}_t + \gamma^{-1}\left(\widehat{B}_t - \frac{T}{B}\int_t^\infty e^{-(\tau-t)r}\mathbb{E}_t[\widehat{T}_\tau]d\tau\right)\end{aligned} \tag{A.11}$$

such that

$$\begin{aligned}\widehat{Y}_t = \rho\int_t^\infty & e^{-(\tau-t)r}\mathbb{E}_t\left[\widehat{Y}_t\right]d\tau + \\ &+ \rho\frac{a}{Y}\widehat{a}_t + \frac{y_1}{Y}\gamma^{-1}\left(\widehat{B}_t - \frac{T}{B}\int_t^\infty e^{-(\tau-t)r}\mathbb{E}_t[\widehat{T}_\tau]d\tau\right) \\ &+ \rho\frac{\mu}{1-\mu}\int_t^\infty e^{-(\tau-t)r}\mathbb{E}_t\left[\frac{T_2}{Y}\widehat{T}_{2t}\right]d\tau - \frac{\mu}{1-\mu}\frac{T_2}{Y}\widehat{T}_{2t}\end{aligned} \tag{A.12}$$

which, since $a_t = B_t$ for the asset market to clear and $r = \rho$, implies

$$\begin{aligned}\widehat{Y}_t =& \rho\int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t[\widehat{Y}_t]d\tau - \rho\gamma^{-1}\frac{y_1}{Y}\int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t[\widehat{T}_\tau]d\tau + \left(\frac{y_1}{Y}\gamma^{-1} + \rho\frac{B}{Y}\right)\widehat{B}_t \\ &+ \rho\frac{\mu}{1-\mu}\int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t\left[\frac{T_2}{Y}\widehat{T}_{2t}\right]d\tau - \frac{\mu}{1-\mu}\frac{T_2}{Y}\widehat{T}_{2t}\end{aligned} \tag{A.13}$$

With a small change in the notation,

$$\begin{aligned}\widehat{Y}_t =& \rho\int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t[\widehat{Y}_\tau]d\tau + \gamma^{-1}\Gamma_1\left(\widehat{B}_t - \rho\int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t[\widehat{T}_\tau]d\tau\right) + \rho\frac{B}{Y}\widehat{B}_t \\ &+ \rho\frac{1}{1-\mu}\int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t\left[\widehat{T}_\tau^{\text{Spender}}\right]d\tau - \frac{1}{1-\mu}\widehat{T}_t^{\text{Spender}}.\end{aligned} \tag{A.14}$$

Here, Γ_1 is the share of income received by saver agents in the steady-state and γ is one over the savers' intertemporal elasticity of substitution. $\widehat{T}_t^{\text{Spender}} = \mu T_2\widehat{T}_{2t}/Y$ is the amount of taxes above their steady-state values *as a percentage of steady-state GDP* levied on spender households in particular. Given an exogenous sequence of tax policy and a corresponding sequence of government debt pinned down by (7) and (3), (A.14) constitutes an intertemporal Keynesian cross of the kind described by Auclert et al. (2024).

The first term of equation (A.14) reflects the MPC of forward-looking savers out of current and future income, ρ. The second term is the effect on aggregate demand from the change in real rates that comes from inflating away current debt and future deficits. The third term represents savers' MPC out of their current liquid assets, times those liquid assets.

The second line of (A.14) depicts how heterogeneity affects the aggregate dynamics of real GDP. The fourth term adjusts for how expected future taxes levied on spender households do not directly affect contemporaneous aggregate demand, even though they come out of households' future incomes. The last term describes the *contemporaneous transfer multiplier* that arises from taxing (or when negative, sending transfers to) spender households. Since the spender households have an MPC of one, the multiplier effect of sending transfers to them is similar to a multiplier for government expenditures. This term is increasing in μ; when spender households make up a larger share of the economy, sending stimulus checks to them (inducing some $-\widehat{T}_t^{\text{Spender}}$) directly increases contemporaneous aggregate demand, expenditures, income, and production.

Appendix A.7. Solving the TANK IKC

Let

$$G_1(t) = \int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t[\widehat{Y}_\tau]d\tau$$

$$G_2(t) = \int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t\left[\frac{\mu T_2}{Y}\widehat{T}_{2\tau}\right]d\tau$$

It follows that

$$G_1'(t) = \frac{d}{dt}\int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t[\widehat{Y}_\tau]d\tau = -\mathbb{E}_t[\widehat{Y}_t] + \int_t^\infty \frac{d}{dt}[e^{-(\tau-t)\rho}\mathbb{E}_t[\widehat{Y}_\tau]]d\tau = -\widehat{Y}_t + \rho G_1(t)$$

$$G_2'(t) = \frac{d}{dt}\int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t\left[\frac{\mu T_2}{Y}\widehat{T}_{2\tau}\right]d\tau = -\frac{\mu T_2}{Y}\mathbb{E}_t[\widehat{T}_{2t}] + \rho G_2(t)$$

And so

$$0 = \overbrace{\rho \int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t[\widehat{Y}_t]d\tau - \widehat{Y}_t}^{G_1'(t)} + \overbrace{-\rho\gamma^{-1}\frac{y_1}{Y}\int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t[\widehat{T}_\tau]d\tau + \left(\frac{y_1}{Y}\gamma^{-1} + \rho\frac{B}{Y}\right)\widehat{B}_t}^{f(t)}$$
$$+ \underbrace{\rho\frac{1}{1-\mu}\int_t^\infty e^{-(\tau-t)\rho}\mathbb{E}_t\left[\frac{\mu T_2}{Y}\widehat{T}_{2t}\right]d\tau - \frac{1}{1-\mu}\frac{\mu T_2}{Y}\widehat{T}_{2t}}_{\frac{1}{1-\mu}G_2'(t)} \tag{A.15}$$

it follows that

$$G_1'(t) = -\frac{\mu}{1-\mu}G_2'(t) + f(t)$$

Integrating from t_0 to ∞,

$$\int_{t_0}^\infty G_1'(t)dt = -\frac{\mu}{1-\mu}\int_{t_0}^\infty G_2'(t)dt + \int_{t_0}^\infty f(t)dt$$

$$\lim_{t\to\infty} G_1(t) - G_1(t_0) = -\left[\frac{1}{1-\mu}\lim_{t\to\infty} G_2(t) - \frac{1}{1-\mu}G_2(t_0)\right] + \int_{t_0}^\infty f(s)ds$$

such that

$$G_1(t) + \frac{1}{1-\mu}G_2(t) + \int_t^\infty f(s)ds = 0$$

Multiplying by ρ and subtracting from the original expression,

$$0 = G_1'(t) - \rho G_1(t) + \frac{1}{1-\mu}[G_2'(t) - \rho G_2(t)] + f(t) - \rho\int_t^\infty f(s)ds$$

$$0 = -\widehat{Y}_t - \frac{1}{1-\mu}\widehat{T}_t^{\text{Spend}} + f(t) - \rho\int_t^\infty f(s)ds$$

such that

$$Y_t = -\frac{1}{1-\mu}\widehat{T}_t^{\text{Spend}} + f(t) - \rho\int_t^\infty f(s)ds$$

From there,

$$\begin{aligned}
\int_t^\infty f(s)ds &= \int_t^\infty \left[-\rho\gamma^{-1}\frac{y_1}{Y}\int_s^\infty e^{-(\tau-s)\rho}\mathbb{E}_s[\widehat{T}_\tau]d\tau + \left(\frac{y_1}{Y}\gamma^{-1}+\rho\frac{B}{Y}\right)\widehat{B}_s\right]ds \\
&= -\rho\gamma^{-1}\frac{y_1}{Y}\int_t^\infty\int_s^\infty e^{-(\tau-s)\rho}\mathbb{E}_s[\widehat{T}_\tau]d\tau ds + \left(\frac{y_1}{Y}\gamma^{-1}+\rho\frac{B}{Y}\right)\int_t^\infty \widehat{B}_s ds \\
&= \gamma^{-1}\frac{y_1}{Y}\int_t^\infty [e^{-(\tau-t)\rho}-1]\mathbb{E}_\tau[\widehat{T}_\tau]d\tau + \left(\frac{y_1}{Y}\gamma^{-1}+\rho\frac{B}{Y}\right)\int_t^\infty \widehat{B}_\tau d\tau
\end{aligned}$$

Simplifying further,

$$\begin{aligned}
Y_t =& -\frac{1}{1-\mu}\widehat{T}_t^{\text{Spend}} - \rho\gamma^{-1}\frac{y_1}{Y}\int_t^\infty [2e^{-(\tau-t)\rho}-1]\mathbb{E}_t[\widehat{T}_\tau]d\tau \\
&+ \left(\frac{y_1}{Y}\gamma^{-1}+\rho\frac{B}{Y}\right)\left(\widehat{B}_t - \rho\int_t^\infty \widehat{B}_\tau d\tau\right)
\end{aligned}$$

Appendix B. Determinacy of the HANK Model Under Different Policy Environments

Both Auclert et al. (2023a) and Hagedorn (2024) propose tests for the uniqueness and determinacy of a linearized rational expectations model using a criterion based on Onatski (2006), which can handle models with theoretically infinite lags and leads. For a model with endogenous states y_t and exogenous states x_t that takes the form

$$\sum_{k=-\infty}^{\infty} A_k \mathbb{E}_t y_{t-k} = \Gamma x_t$$

Onatski (2006) proposes constructing the complex-valued criterion function

$$\det \widehat{A}(\lambda) = \det \left[\sum_{k=-\infty}^{\infty} A_k e^{ik\lambda} \right] \tag{B.1}$$

where $i = \sqrt{-i}$ is Euler's constant and k is the number of lags (such that the coefficients are presented going back in time relative to t). As such, $\widehat{A}(\lambda)$ is essentially the discrete[6] Fourier transform of the model's time indexed matrix coefficients, and so describes the phase and amplitude of different frequencies $\lambda \in [0, 2\pi]$ that generate the coefficients. He then defines the *winding number* of the criterion function as the contour integral of the function evaluated

[6] For a continuous time system, the analogous model would be

$$\int_{-\infty}^{\infty} A_\tau \mathbb{E}_t x_{t-\tau} d\tau = \Gamma z_t$$

and a criterion that uses the continuous Fourier transform

$$\widehat{A}(\lambda) = \int_{-\infty}^{\infty} A_\tau e^{i\lambda\tau} d\tau.$$

However because my numerical solution of the model is discretized for time grid points of a fixed interval, this is essentially tantamount to using the discrete formulation, but with the rotation re-scaled by the size of the time step Δt, as $t = \Delta t \times k$, such that the criterion function becomes

$$\det \widehat{A}(\lambda) = \det \left[\sum_{k=-\infty}^{\infty} A_k e^{ik\lambda\Delta t} \Delta t \right].$$

Since the sequence space numerical solution of my model is essentially a discrete-time system on the computer, I evaluate its Onatski (2006) criterion as one would a discrete-time model.

over $[0, 2\pi]$ – tantamount to evaluating the Z (Laplace) transformation of the coefficients over the unit circle in the complex plane – which quantifies how many times the graph of the function encircles the origin. For a large class of economic models the author terms "generic,"[7] the model has a unique solution if the winding number is zero such that the graph of the criterion function from $[0, 2\pi]$ does not enclose the origin.

As Auclert (2018) discusses, the intuition is similar to that of the Blanchard and Kahn (1980) conditions. If the winding number is equal to zero, then the criterion function has as many zeros as poles outside of the unit circle via the Cauchy argument principle, and therefore essentially has as many explosive roots as non-predetermined variables. If the function wraps around the origin counter-clockwise (such that it has a positive winding number) then the model has no solution; if it wraps around the origin counter-clockwise (such that it has a negative winding number), then there exist a multiplicity of solutions.

However, the original Onatski (2006) criterion was designed for time-invariant systems, where only the difference in time determined the system's interaction with its own leads and lags. For the sequence-space Jacobian method proposed by Auclert et al. (2021), this requires that the sequence space Jacobian matrix is Toeplitz, a property that it does not generally have. However, Auclert et al. (2023a) note that HANK models typically have "quasi"-Toeplitz structure, in that the response of the system at time t to a future perfect foresight shock at time s becomes largely invariant to the precise date s and instead only depends on $s - t$. Different future shocks, in other words, begin to look like time-transposed versions of one another. Auclert et al. (2023a) then argue that they can approximate

$$A_k = \lim_{t \to \infty} A_{k,t}$$

where A_k are the elements of the sequence-space Jacobian matrix that the endogenous states at time t to their values k periods in the past. The authors then impose the Onatski (2006) criterion on the system's response to a future shock and argue that it provides a check for the determinacy of the system overall.

[7]Onatski (2006) defines models as "generic" where all of the time shift components of the Wiener-Hopf factorization of the criterion, called partial indexes, are either zero or of the same sign.

Hagedorn (2023) takes a similar, but slightly different, approach. The author employs a dimension reduction routine to the equilibrium and models the economy such that agents do not track the whole distribution, but instead only track the aggregate level of assets. In doing so, the agents forecast prices in the economy under the assumption that the future distribution looks like the steady state one – but with all of the other agents' wealth scaled up or down by the aggregate asset position. If the aggregate asset position is included as a state variable, the simplified system becomes truly Toeplitz – such that the Onatski (2006) criterion may be straightforwardly applied.

Lastly, Bayer and Luetticke (2020) uses a completely different numerical approach and suggests solving HANK models in state-space using a dimension reduction strategy similar to the one employed by Reiter (2009). In the last section of this appendix, I detail the steps and how it may be used to solve my HANK model. They argue that the dimension-reduced model's stability and determinacy may then be evaluated as in Blanchard and Kahn (1980): a system has a unique solution if it has as many explosive (positive) eigenvalues as it has jump variables. I solve my state space model using the QZ decomposition suggested by Sims (2002) and consider its generalized eigenvalues.

In addition to the active fiscal, passive monetary HANK calibration explored in the main paper, I consider the determinacy of a passive fiscal, passive monetary calibration as well. I do this by setting the automatic debt repayment parameter of the HANK model to $\kappa = 0.01$. Since this κ is double the steady-state real interest rate $r_n ss = 0.005$, debt converges back to its steady-state values even in the absence of inflation, so the fiscal policy is indeed "passive." For both settings settings, I check the determinacy of my model in all three ways. The Bayer and Luetticke (2020) results are straightforward; my dimension-reduced system has as many explosive eigenvalues as it has forward-looking control variables. The graphs of the criterion functions for both the Auclert et al. (2023a) and Hagedorn (2023) methodologies are displayed in Figure B.7. None of the graphs encircle the origin.

All three different methodologies suggest that in each of my HANK calibrations, the model exhibits local determinacy.

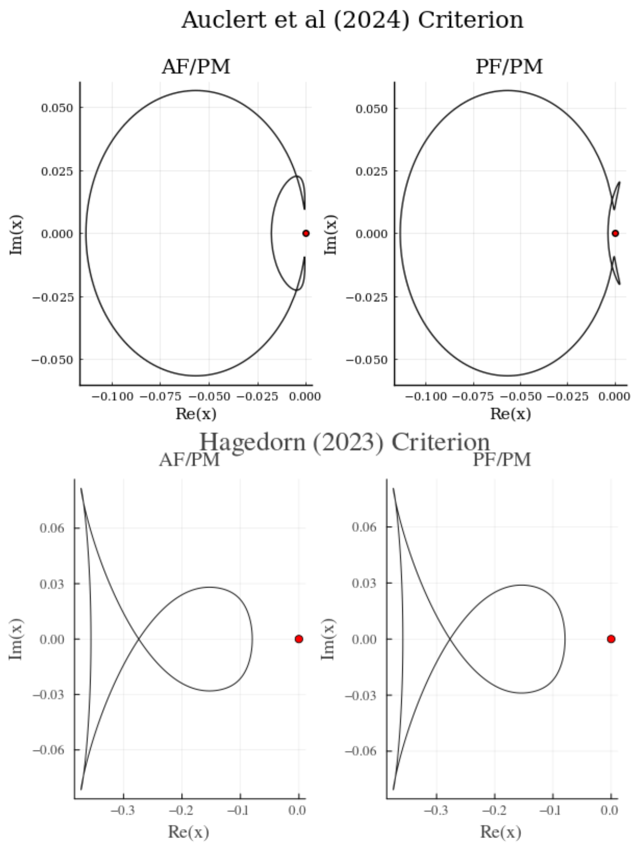

Figure B.7: Onatski criterion for a sequence-space solution of a HANK model, both with the Auclert et al. (2023a) determinacy criterion (top row) and the Hagedorn (2023) criterion (bottom row). Active fiscal, passive monetary policy criterion plots are on the left, while the plots on the right depict a passive/passive configuration. None of the criterion wind around the origin, implying that the model has a unique solution under the different calibrations listed in Table **??**. Arrows denote the direction of the graph around the origin.

Appendix B.1. Brief Summary of Intuition Provided by Auclert et al. (2023a)

To briefly sketch the intuition of Onatski (2006)'s methodology, Auclert et al. (2023a) note that Onatski (2006) essentially recommends taking determinant of the z-transformation (discrete-time Laplace transformation) of the sequence of the model's coefficients, a common technique used in signal processing:

$$\det \widehat{A}(z) = \det \left[\sum_{k=-\infty}^{\infty} A_k z^k \right]$$

where $z = e^{\alpha + i\omega} \in \mathbb{C}$ is a point in the complex plane that describes both a sinusoidal frequency and exponential magnitude. As noted in Auclert et al. (2023a), the contour integral of the graph of $\det \widehat{A}(z)$ evaluated over the unit circle $|z| = 1$ is known as the function's *winding number*, as it counts the number of times the function wraps around the origin counter-clockwise. They further denote the number of zeros of $\det \widehat{A}(z)$ inside the unit circle as N; these are essentially stable roots. r predetermined variables affect the current state in the z-transformation via a time shift of z^{-r}; with the fundamental theory of algebra, z^r has r roots, such that the criterion function then has r stable poles. They then note that via Cauchy's argument principle,

$$\operatorname{wind}\det \widehat{A}(z) = \frac{1}{2\pi i}\oint_{\det \widehat{A}(C)} \frac{dz}{z} = N - r$$

If $\det \widehat{A}(z)$ does not wrap around zero, then $Z - P = 0$ and the number of zeros in the unit circle is equal to the number of poles, and the system admits a unique solution. As a corollary, the number of stable roots is equal to the number of predetermined state variables, matching the Blanchard and Kahn (1980) conditions for existence and determinacy.

Appendix B.2. Onatski (2006) and Partial Indexes

Onatski (2006) constructs his criterion using the Wiener-Hopf factorization of $\widehat{A}(\lambda)$ into three components: an explosive root component $\widehat{A}_+(\lambda)$, a stable component $\widehat{A}_-(\lambda)$, and a component that only pertains to the time shift of the coefficients (which can be accomplished by multiplication or division of the z-transform by a factor of $e^{\lambda i}$) $A_0(\lambda)$. All together,

$$\widehat{A}(\lambda) = \widehat{A}_-(\lambda)\widehat{A}_0(\lambda)\widehat{A}_+(\lambda)$$

He notes that the time shift component $\widehat{A}_0(\lambda)$ is a diagonal matrix $\operatorname{diag}(e^{i\lambda k_1}, \ldots, {}^{i\lambda k_n})$, where n is the number of variables in x_t and $(k_1, \ldots, k_n)$ are the number of periods each variable is lagged time shift component of the factorization, known as the "partial indexes." Onatski (2006) calls a model "generic" if its winding numbers are all of the same sign or zero. Then, a winding number of zero implies that all of the partial indexes of the model are zero as well.

From his paper, Proposition 1 then states that if the partial indexes are all i) equal to zero, then the model solution exists and is unique, ii) weakly negative, with at least one strictly negative, then the model is indeterminate, and iii) weakly positive, with at least one strictly positive, then a solution does not exist. He then shows that, because the winding number of the root components is always zero and the time shift matrix containing the partial indexes is diagonal,

$$\begin{aligned}
\operatorname{wind}\det\widehat{A}(\lambda) =& \operatorname{wind}(\det[\widehat{A}_{-}(\lambda)]\det[\widehat{A}_{0}(\lambda)]\det[\widehat{A}_{+}(\lambda)] \\
=& \operatorname{wind}\det[\widehat{A}_{-}(\lambda)] + \operatorname{wind}\det[\widehat{A}_{0}(\lambda) + \operatorname{wind}\det[\widehat{A}_{+}(\lambda)] \\
=& \operatorname{wind}\det\widehat{A}_{0}(\lambda) \\
=& \operatorname{wind}\exp\left(\lambda i\sum_{j=1}^{T}k_j\right) = \sum_{j=1}^{T}k_j
\end{aligned}$$

The winding number is equal to the sum of partial indexes. Thus, if a model is generic, then the winding number will only be equal to zero if the partial indexes are all zero, negative if the partial indexes are all weakly negative, and positive if the partial indexes are all weakly positive.

Appendix C. HANK Model Derivations

Appendix C.1. Wage Phillips Curve

This is a continuous-time version of Auclert et al. (2024), *The Intertemporal Keynesian Cross*. Say a labor-aggregator hires labor from households to create an aggregate unit of input labor:

$$L_{k,t} = \int_0^1 (z_i h_{ikt}) di$$

And labor from each union is differentiated with elasticity of substitution ε_ℓ:

$$L_t = \left(\int_0^1 L_{k,t}^{\frac{\varepsilon_\ell - 1}{\varepsilon_\ell}} dk \right)^{\frac{\varepsilon_\ell}{\varepsilon_\ell - 1}}$$

Let W_t be the nominal wage paid by employers to labor-aggregators, and let the labor-aggregator pay its workers a nominal wage of $W_{k,t}$. Labor-aggregating firms thus hire according to

$$\max_{\{L_{k,t}\}_{k \in [0,1]}} W_t \left(\int_0^1 L_{k,t}^{\frac{\varepsilon_\ell - 1}{\varepsilon_\ell}} dk \right)^{\frac{\varepsilon_\ell}{\varepsilon_\ell - 1}} - \int_0^1 W_{k,t} L_{k,t} dk$$

such that from the FOCs, the demand for labor from union k is

$$W_t \left(\int_0^1 L_{k,t}^{\frac{\varepsilon_\ell - 1}{\varepsilon_\ell}} dk \right)^{\frac{\varepsilon_\ell}{\varepsilon_\ell - 1} - 1} L_{k,t}^{-\frac{1}{\varepsilon_\ell}} - W_{k,t} = 0$$

$$W_t L_t^{\frac{1}{\varepsilon_\ell}} L_{k,t}^{-\frac{1}{\varepsilon_\ell}} = W_{k,t}$$

$$W_t L_t^{\frac{1}{\varepsilon_\ell}} = W_{k,t} L_{k,t}^{\frac{1}{\varepsilon_\ell}}$$

$$\Rightarrow \quad \frac{L_{k,t}}{L_t} = \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell}$$

Unions face nominal wage adjustment costs:

$$\frac{\theta_w}{2} \int_0^1 \pi_{w,k}^2 dk, \quad \text{where} \quad \pi_{w,k} = \frac{dW_{k,t}}{dt} \frac{1}{W_{k,t}}$$

The labor union k sets wages to maximize its members' lifetime utilities:

$$J_t^w(W_{k,t}) = \max_{\pi_{k,t}^w} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\int \int \left\{ \frac{c(a,z)^{1-\gamma}}{1-\gamma} - \frac{h(a,z)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\} \mu_t(a,z) da\ dz - \frac{\theta_w}{2}(\pi_{k,t}^w)^2 \right] dt$$

$$\text{s.t. } \frac{dW_t}{dt} = \pi_t^w W_t$$

$$L_{k,t} = \int_0^1 z_i h_{ikt} di$$

$$\frac{L_{k,t}}{L_t} = \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell}$$

Where the third equation follows from the first-order conditions from the households.

The HJB is then (suppressing the value function's arguments for brevity)

$$\rho J_t^w = \left[\int \int \left\{ \frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma} - \frac{h(a,z;W_{k,t})^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\} \mu_t(a,z) da\ dz - \frac{\theta_w}{2}(\pi_{k,t}^w)^2 \right] + \frac{\partial J_t^w}{\partial W_{k,t}} \pi_t^w W_{k,t} + \frac{\partial J_t^w}{\partial t}$$

The FOC for wage inflation is then

$$-\theta_w \pi_{k,t}^w + \frac{\partial J^w(W_{k,t})}{\partial W_{k,t}} W_{k,t} = 0$$

$$\Rightarrow \quad \frac{\partial J^w(W_{k,t})}{\partial W_{k,t}} = \theta_w \frac{\pi_{k,t}^w}{W_{k,t}}$$

Taking the total differential of the marginal value of wages,

$$d\left(\frac{\partial J_t^w(W_{k,t})}{\partial W_{k,t}} \right) = \partial_{W_{k,t}}^2 J_t^w dW_{k,t} + \partial_{W_{k,t}} \partial_t J_t^w dt$$

and doing the same to the LHS of the wage inflation FOC,

$$d\left(\theta_w \frac{\pi_t^w}{W_{k,t}} \right) = \frac{\theta_w}{W_{k,t}} d\pi_t^w - \frac{\theta_w \pi_t^w}{W_{k,t}^2} dW_{k,t}$$

I can equate the two:

$$\frac{\theta_w}{W_{k,t}} d\pi_t^w - \frac{\theta_w \pi_t^w}{W_{k,t}^2} dW_{k,t} = \partial_{W_{k,t}}^2 J^w dW_{k,t} + \partial_t \partial_{W_{k,t}} J_t^w dt.$$

Taking expectations and dividing by dt yields

$$\frac{\theta_w}{W_{k,t}}\frac{\mathbb{E}_t[d\pi_t^w]}{dt}-\frac{\theta_w\pi_t^w}{W_{k,t}}\underbrace{\frac{dW_{k,t}}{dt}\frac{1}{W_{k,t}}}_{\pi_{k,t}^w}=\partial^2_{W_{k,t}}J_t^w\frac{dW_{k,t}}{dt}+\partial_{W_{k,t}}\partial_t J_t^w$$

such that

$$\frac{\theta_w}{W_{k,t}}\frac{\mathbb{E}_t[d\pi_t^w]}{dt}-\frac{\theta_w\pi_t^w}{W_{k,t}}\pi_t^w=\partial^2_{W_{k,t}}J^w\pi_t^w W_{tk}+\partial_{W_{k,t}}\partial_t J_t^w \tag{C.1}$$

Next, the Envelope condition stipulates that

$$\begin{aligned}\rho\partial_{W_{k,t}}J_t^w=&\left[\int\int\partial_{W_{k,t}}\left\{\frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma}-\frac{h(a,z;W_{k,t})^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right\}\mu_t(a,z)da\ dz\right]\\&+\partial^2_{W_{k,t}}J^w\pi_t^w W_{k,t}+\partial_{W_{k,t}}J^w(W_{k,t})\pi_t^w+\partial_{W_{k,t}}\partial_t J_t^w\end{aligned}$$

Substituting in (C.1),

$$\begin{aligned}\rho\partial_{W_{k,t}}J_t^w=&\left[\int\int\partial_{W_{k,t}}\left\{\frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma}-\frac{h(a,z;W_{k,t})^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right\}\mu_t(a,z)da\ dz\right]\\&+\partial_{W_{k,t}}J_t^w(W_{k,t})\pi_t^w+\frac{\theta_w}{W_{k,t}}\frac{\mathbb{E}_t[d\pi_t^w]}{dt}-\frac{\theta_w\pi_t^w}{W_{k,t}}\pi_t^w\end{aligned}$$

and then the FOC,

$$\begin{aligned}\rho\theta_w\frac{\pi_{k,t}^w}{W_{k,t}}=&\left[\int\int\partial_{W_{k,t}}\left\{\frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma}-\frac{h(a,z;W_{k,t})^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right\}\mu_t(a,z)da\ dz\right]\\&+\theta_w\frac{\pi_{k,t}^w}{W_{k,t}}\pi_t^w+\frac{\theta_w}{W_{k,t}}\frac{\mathbb{E}_t[d\pi_t^w]}{dt}-\frac{\theta_w\pi_t^w}{W_{k,t}}\pi_t^w\end{aligned}$$

it follows that

$$\rho\pi_{k,t}^w=\frac{W_{k,t}}{\theta_w}\left[\int\int\partial_{W_{k,t}}\left\{\frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma}-\frac{h(a,z;W_{k,t})^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right\}\mu_t(a,z)da\ dz\right]+\frac{\mathbb{E}_t[d\pi_t^w]}{dt}. \tag{C.2}$$

From the households' envelope condition, the change in utility from wages will be equal to the marginal utility, times the change in earnings:

$$\partial_{W_{k,t}} \left\{ \frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma} \right\} = c(a,z)^{-\gamma}(1-\tau)\partial_{W_{k,t}} \left(z \frac{W_{k,t}}{P_t} h(a,z) \right)$$

Where if households uniformly supply their labor to union k, and unions internalize their labor's demand:

$$h_{ikt}(a,z) = \frac{1}{Z} L_{k,t} = \frac{1}{Z} \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell} L_t$$

$$\begin{aligned}
\Rightarrow \quad \partial_{W_{k,t}} \left\{ \frac{c(a,z;W_{k,t})^{1-\gamma}}{1-\gamma} \right\} &= c(a,z)^{-\gamma}(1-\tau)\partial_{W_{k,t}} \left(z \frac{W_{k,t}}{P_t} \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell} \frac{1}{Z} L_t \right) \\
&= c(a,z)^{-\gamma}(1-\tau)(1-\varepsilon_\ell) \frac{z}{Z} \frac{1}{W_{k,t}} \left(\frac{W_{k,t}}{P_t} \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell} L_t \right) \\
&= c(a,z)^{-\gamma}(1-\tau)(1-\varepsilon_\ell) \frac{z}{Z} \frac{1}{P_t} L_{k,t}
\end{aligned}$$

For the effect of wages on labor disutility, I can directly evaluate

$$\partial_{W_{k,t}} h(a,z) = \frac{1}{Z} \partial_{W_{k,t}} \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell} L_t = -\varepsilon_\ell \frac{1}{Z} \frac{1}{W_{k,t}} \left(\frac{W_t}{W_{k,t}} \right)^{\varepsilon_\ell} L_t = -\varepsilon_\ell \frac{1}{Z} \frac{L_{k,t}}{W_{k,t}}$$

Plugging in the results into (C.2),

$$\rho \pi^w_{k,t} = \frac{W_{k,t}}{\theta_w} \left[\int \int \left\{ c(a,z)^{-\gamma}(1-\tau)(1-\varepsilon_\ell) \frac{z}{Z} \frac{1}{P_t} L_{k,t} + \frac{1}{Z} h(a,z)^{\frac{1}{\eta}} \varepsilon_\ell \frac{L_{k,t}}{W_{k,t}} \right\} \mu_t(a,z) da \ dz \right] + \frac{\mathbb{E}_t[d\pi^w_t]}{dt}$$

$$\rho \pi^w_{k,t} = \frac{\varepsilon_\ell}{\theta_w} \frac{L_{k,t}}{Z} \int \int \left\{ h(a,z)^{\frac{1}{\eta}} - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell} (1-\tau) z \frac{W_{k,t}}{P_t} c(a,z)^{-\gamma} \right\} \mu_t(a,z) da \ dz + \frac{\mathbb{E}_t[d\pi^w_t]}{dt}$$

Leading to the wage Phillips Curve

$$\frac{\mathbb{E}_t[d\pi^w_t]}{dt} = \rho \pi^w_t - \frac{\varepsilon_\ell}{\theta_w} \frac{L_t}{Z} \int \int \left(h(a,z)^{\frac{1}{\eta}} - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell} (1-\tau) z w_t c(a,z)^{-\gamma} \right) \mu_t(a,z) \ da \ dz \quad \text{(C.3)}$$

where $w_t \equiv \frac{W_{k,t}}{P_t}$ is the real wage in the symmetric equilibrium where $W_{k,t} = W_t \ \forall k \in [0,1]$.

Log-linearizing for a representative agent,

$$\frac{\mathbb{E}_t[d\pi^w_t]}{dt} = \rho \pi^w_t - \frac{\varepsilon_\ell}{\theta_w} \frac{Y_t}{Z} \left[\left(\frac{Y_t}{Z} \right)^{\frac{1}{\eta}} - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell} (1-\tau) Z w Y_t^{-\gamma} \right] \quad \text{(C.4)}$$

$$\frac{\mathbb{E}_t[d\pi_t^w]}{dt} = \rho\pi_t^w - \frac{\varepsilon_\ell}{\theta_w}\frac{Y(1+\widehat{Y}_t)}{Z}\left[\left(\frac{Y}{Z}\right)^{\frac{1}{\eta}}(1+\frac{1}{\eta}\widehat{Y}_t) - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)ZwY^{-\gamma}(1-\gamma\widehat{Y}_t)\right] \quad \text{(C.5)}$$

$$\frac{\mathbb{E}_t[d\pi_t^w]}{dt} = \rho\pi_t^w - \frac{\varepsilon_\ell}{\theta_w}\frac{Y(1+\widehat{Y}_t)}{Z}\left[\left(\frac{Y}{Z}\right)^{\frac{1}{\eta}}\frac{1}{\eta}\widehat{Y}_t + \frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)Zw\gamma Y^{-\gamma}\widehat{Y}_t\right] \quad \text{(C.6)}$$

$$\frac{\mathbb{E}_t[d\pi_t^w]}{dt} = \rho\pi_t^w - \frac{\varepsilon_\ell}{\theta_w}\frac{Y}{Z}\left[\left(\frac{Y}{Z}\right)^{\frac{1}{\eta}}\frac{1}{\eta} + \gamma\frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)ZwY^{-\gamma}\right]\widehat{Y}_t \quad \text{(C.7)}$$

Note that this implies a Phillips Curve slope of the NKPC with respect to output is roughly 0.275, given the proposed parameters. If the slope is measured as just the component that relates to increases in marginal labor disutility, however, the slope is $\frac{\varepsilon_\ell}{\theta_w}\frac{Y_t}{Z}\left(\frac{Y_t}{Z}\right)^{\frac{1}{\eta}} = 0.07$.

Appendix D. Robustness

Appendix D.1. Present Value Sacrifice Ratios

Instead of the definitions presented in the main paper, one could instead construct sacrifice ratios that reflect the cumulative net present value of output gaps and inflation:

$$\mathcal{C}Y_t^{PV} \equiv \int_0^t e^{-rt}\widehat{Y}_s ds$$

$$\mathcal{C}\pi_t^{PV} \equiv \int_0^t e^{-rt}\pi_s ds.$$

Here, r is the real interest rate in the non-stochastic steady-state. The alternative price level sacrifice ratio is then

$$SR_t^{PV} \equiv \frac{(\mathcal{C}Y_t^{PV}/4)}{\mathcal{C}\pi_t^{PV}}.$$

One could then examine how the present values of inflation and output evolve (from the perspective of an individual at time 0) for the policies that send a 1% of GDP transfer to different households. However, the resulting plot almost exactly matches Figure 3 in section 5.

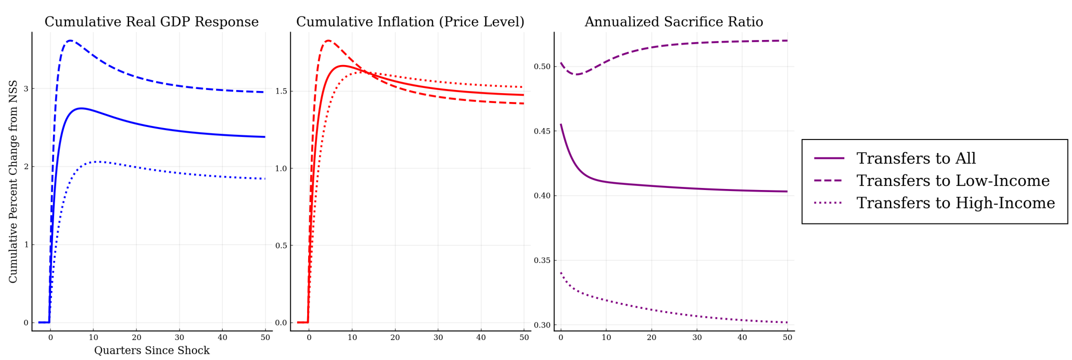

Figure D.8: Present value cumulative output gaps, inflation, and price-level sacrifice ratios for 1% of GDP fiscal transfers sent to low-income, high-income, and all households.

However, instead of the percentage difference between the eventual price level in the low-income transfer scenario and the untargeted scenario being 5% of the latter, the percentage difference shrinks to 3.8%.

Appendix D.2. The Slope of the Phillips Curve

In the main parameterization, I set $\frac{\varepsilon}{\theta_\pi} = 0.10$, where $\theta_\pi = 100$. Mapping this to a discrete-time quarterly Calvo pricing model, if α is the percentage of unions that do not adjust their prices,

$$\frac{(1-\alpha)(1-\alpha e^{-\rho})}{\alpha} = 0.10 \;\Rightarrow\; \alpha = 0.74$$

such that roughly 26% of wage contracts reset every quarter and the average wage contract resets in slightly under a year.

Below in Figure D.9, I plot the cumulative impulse responses in the active fiscal/passive monetary regime under different parameterizations with different degrees of nominal rigidity. The main calibration, $\theta_\pi = 100$, is plotted with a solid line. Decreasing θ_π to 50 amounts to lowering nominal rigidities and doubling the slope of the Phillips curve, while doubling it to 200 is tantamount to halving the curve's slope.

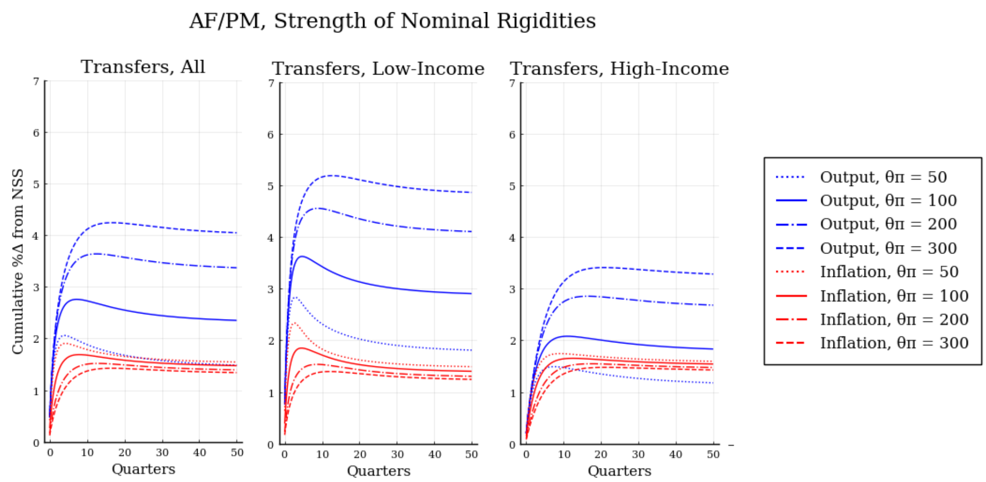

Figure D.9: Cumulative impulse response of output and inflation with different degrees of nominal rigidity. $\theta_\pi = 100$ is the baseline specification. Doubling θ_π halves the slope of the Phillips curve.

More nominal rigidities (and a flatter Phillips Curve) amplify the output response and smooth the path of inflation, while decreasing nominal rigidities does the converse. Even so, the price level eventually settles to roughly the same value in each experiment and parameterization. The ordering of the fiscal responses is also unchanged, even though their magnitudes are altered.

The implied cumulative sacrifice ratios (cumulative output gaps as a percentage of annual GDP divided by cumulative inflation) are plotted in Figure D.10. Lowering nominal rigidities

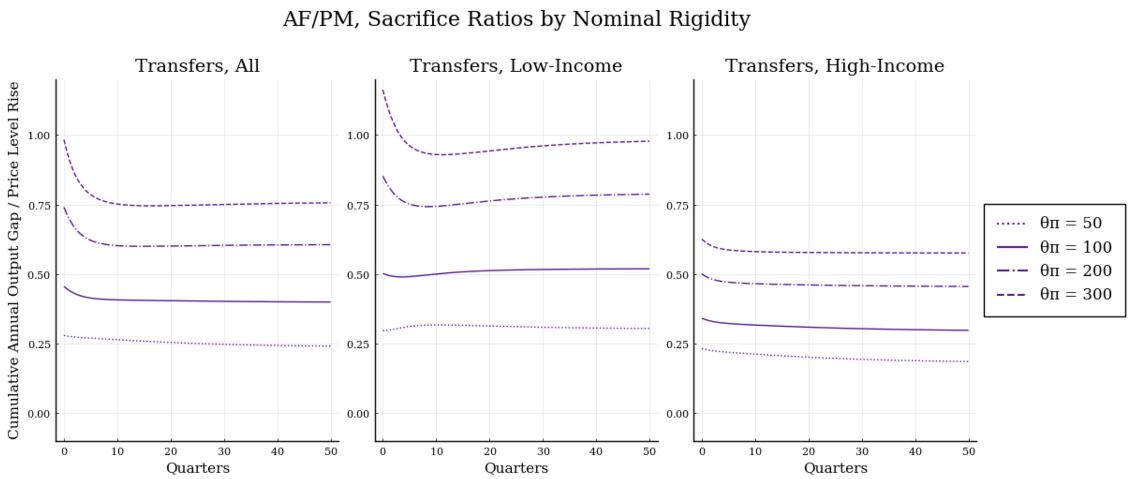

Figure D.10: Price level sacrifice ratios by transfer type and nominal rigidity.

compresses the difference in sacrifice ratios across policies, while increasing them increases the dispersion. Even so, the relative ordering between policies with the same parameterization is unchanged. Sacrifice ratios are lower for GDP changes induced by reductions in transfers to above-average income households, and are higher transfers to those with below-average income are reduced.

Appendix D.3. Firm Profits

In previous drafts of this paper, I included specifications where intermediate firms had a constant markup of $\varepsilon/(\varepsilon - 1)$, where profits were distributed proportionally to labor income z. In my main specification, I set $\varepsilon \to \infty$, effectively making the intermediate firm sector perfectly competitive and removing firm profits from the model entirely.

In many HANK models, the distribution and cyclicality of firm profits can substantially affect the simulated dynamics. However, the inclusion or exclusion of these profits has little effect on my paper's conclusions. I plot cumulative impulse response functions for the active fiscal/passive monetary regime in Figure D.11, varying the elasticity of substitution of the output of intermediate firms ε as I do so. $\varepsilon = 7$ corresponds to profits composing 14% of national income, while $\varepsilon = 20$ reduces them to 5%. The main calibration of $\varepsilon \to \infty$ is plotted with solid lines.

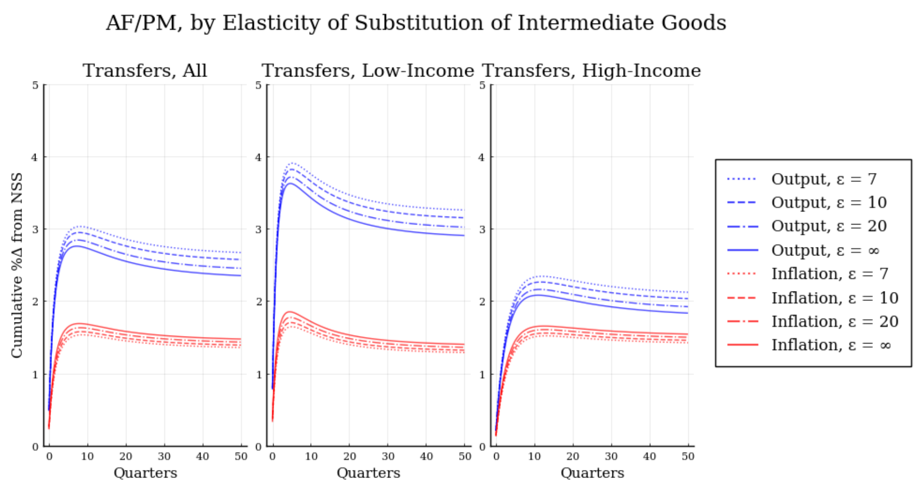

Figure D.11: Model solutions with different shares of profit income, $1/\varepsilon$.

Since I use a sticky-wage model with perfectly flexible output prices, however, firm markups are completely acyclic. Profit income thus only fluctuates due to changes in aggregate output, which are small when multiplied by the profit share. This profit income is further distributed proportionally to z to on-average wealthier agents with lower MPCs, so its effect on the model dynamics is small even when the profit share is realistically calibrated. As such, to avoid questions of the distribution of profits, I drop them from the model entirely.

Appendix E. Solving Bayer and Luetticke (2020) in Continuous Time

This section is best viewed after having already read Achdou et al. (2021), Ahn et al. (2018), and particularly Bayer and Luetticke (2020) as background; the below section largely amounts to a brief sketch of adapting Bayer and Luetticke (2020) to continuous time. For notational brevity, I write the infinitessimal generator operator of the concentrated Hamilton Jacobi Bellman equation as

$$\begin{aligned}\mathcal{D}[V] =& \lim_{t\downarrow 0} \frac{\mathbb{E}_t^{a,z}[V_t(a_{t+dt}, z_{t+dt})] - V_t(a_t, z_t)}{dt} \\ =& \frac{\partial V_t}{\partial a}(a,z)\frac{q_{NSS}}{q_t}\left[(1-\tau)w_t z h_t(a,z) + M_t(z_t;\zeta_t) - c + \left(r_t - \frac{dq_t}{dt}\frac{1}{q_t}\right)\frac{q_t}{q_{NSS}}a\right] \\ &+ \frac{\partial V_t}{\partial z}(a,z;\mu,\zeta)z\left[\frac{1}{2}\sigma_z^2 - \theta_z \log(z)\right]\end{aligned}$$

where the expectation operator is taken with respect to only the idiosyncratic variables. As in Achdou et al. (2021), I write the adjoint operator (which describes the Kolmogorov forward equation of the idiosyncratic state distribution) as $\mathcal{D}^*$, where the KFE operator is the adjoint of the maximized HJB operator in L^2 space. Additionally, I write expectation errors for a jump variable "J" as $d\delta_{J,t}$, such that $d\delta_{J,t} = dJ_t - \mathbb{E}_t[dJ_t]$.

Suppose aggregate shocks in the economy evolve according to

$$d\zeta_t = -\Theta_\zeta \zeta_t dt + d\epsilon_{\zeta,t}. \tag{E.1}$$

A sequential equilibrium following a perturbation from the steady state $W_{\zeta,0}$ is a resulting path of aggregate shocks $\{\zeta_t\}_{t\geq 0}$, a series of value functions $\{V_t(a,z)\}_{t\geq 0}$, consumption decisions and labor allocations $\{c_t(a,z), h_t(a,z)\}_{t\geq 0}$, distributions $\{\mu_t(a,z)\}_{t\geq 0}$, outstanding government debt $\{B_t\}_{t\geq 0}$, wages $\{w_t\}_{t\geq 0}$, nominal and real interest rates $\{i_t, r_t\}_{t\geq 0}$, bond prices $\{q_t\}_{t\geq 0}$, and inflation rates $\{\pi_t\}_{t\geq 0}$ where

$$dV_t(a,z) = \left\{\rho V_t(a,z) - \left[u(c_t(a,z)) - v(h_t(a,z)) + \mathcal{D}[V]\right]\right\}dt - \frac{\partial V_t(a,z)}{\partial a}d\delta_{qB,t} + d\delta_{V(a,z),t} \tag{E.2}$$

and if $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$, it follows that the FOC for consumption is

$$c_t(a,z)^{-\gamma} = \frac{\partial V_t}{\partial a}(a,z). \tag{E.3}$$

The distribution evolves according to

$$d\mu_t(a,z) = \mathcal{D}^*[\mu]dt \tag{E.4}$$

while labor is supplied to meet market demand:

$$h_t(a,z) = \frac{L_t}{Z} \tag{E.5}$$

Inflation is equal to nominal wage inflation, which follows the labor market Phillips Curve

$$d\pi_t^w = \left\{\rho\pi_t^w - \frac{\varepsilon_\ell}{\theta_w}L_t \int\int \left(v'(h(a,z)) - \frac{\varepsilon_\ell - 1}{\varepsilon_\ell}(1-\tau)zw_t u'(c(a,z))\right) da\ dz\right\} dt + d\delta_{\pi^w,t} \tag{E.6}$$

Real wages are then constant:

$$w_t = w_{\text{NSS}} = \frac{\varepsilon - 1}{\varepsilon} \tag{E.7}$$

where ε is the elasticity of substitution between goods in the output sector; the profit-free version of the model sets $\varepsilon \to \infty$. The government's budget constraint must satisfy

$$dB_t = -(T_t - G_t)dt + r_t B_t dt + \frac{d\delta_{qB,t}}{q_t}B_t \tag{E.8}$$

where nominal bond prices and equity prices satisfy

$$dq_t = q_t\left(i_t + \omega - \frac{\omega}{q_t}\right)dt + d\delta_{q,t} \tag{E.9}$$

Equilibrium must also be consistent with the Fisher equation, the marginal cost equation, and the profit equation:

$$r_t = i_t - \pi_t \tag{E.10}$$

$$m_t = w_t \tag{E.11}$$

$$\Pi_t = [1 - m_t]Y_t \tag{E.12}$$

All goods consumed must be produced:

$$Y_t = L_t \tag{E.13}$$

and the idiosyncratic variables must aggregate:

$$C_t = \int_0^\infty \int_{\underline{a}}^\infty c_t(a,z)\mu_t(a,z)da\ dz \tag{E.14}$$

$$L_t = \int_0^\infty \int_{\underline{a}}^\infty z h_t(a,z)\mu_t(a,z)da\ dz \tag{E.15}$$

Finally, goods and financial markets must clear:

$$Y_t = C_t \tag{E.16}$$

$$B_t = \int_0^\infty \int_{\underline{a}}^\infty a\mu_t(a,z)da\ dz \tag{E.17}$$

I write the vector of forward-looking control variables as

$$X_{C,t} = (V_t(a,z),\ \pi_t,\ q_t)',$$

the set of state variables as

$$X_{1,t} = (\mu_t(a,z),\ B_t, \zeta_t)',$$

and the vector of static constraints as

$$X_{L,t} = (Y_t,\ L_t)',$$

(where many of the static constraints like the Fisher equation and the employment rules can be re-written to solve out the other static variables from the model). Stacking the controls, states, and static variables, I write

$$X_t = (X_{C,t}, X_{1,t}, X_{L,t})'$$

where dX_t represents the differentials of X_t. Using this succinct notation, the entire system (E.1-E.17) can be written as

$$\Gamma_0 dX_t = \Omega(X_t, d\delta_{X,t}, d\epsilon_{\zeta,t}) \tag{E.18}$$

where the rows of Γ_0 corresponding to static constraints are equal to zero.

I discretize the partial differential equations on the computer in the non-stochastic steady state where $X_t = X_{NSS}$, $dX_t = 0$, $d\delta_{X,t} = 0$, and $d\epsilon_{\zeta,t} = 0$, using the finite-differences methodology described in Achdou et al. (2021). This entails discretizing (E.18) via an upwind finite difference approximation for the partial derivatives along an asset grid (which I index by $i \in I \equiv \{1, \ldots, N_a\}$) and an income grid (which I index by $j \in J \equiv \{1 \ldots, N_z\}$). The tensor $V_{i,j,nss}$ then approximates the value function $V_{NSS}(a_i, z_j)$ in the discretized state space, while the tensor $\mu_{i,j,nss}$ approximates the distribution $\mu_{NSS}(a_i, z_j)$.

Before proceeding, I find it useful to define $\widehat{X}_t \equiv X_t - X_{NSS}$ as either the level deviations or the log deviations of the variables from their values in the non-stochastic steady state. As such, the complete system can be rewritten to become

$$\Gamma_0 d\widehat{X}_t = \widehat{\Omega}(\widehat{X}_t, d\delta_{X,t}, d\epsilon_{\zeta,t}) \tag{E.19}$$

where the arguments are the deviation terms. The steady state thus satisfies $\widehat{\Omega}(\mathbf{0}) = \mathbf{0}$. I then proceed to solve for the dynamics of the economy following aggregate shocks. Practically, the dimensionality of the discretized value functions and distributions necessitate dimension reduction. However, for clarity, I first describe the process *without* dimension reduction.

Appendix E.1. Without Dimension Reduction

With the non-stochastic steady state (NSS) in hand, I then calculate the numerical Jacobian of the system at the NSS using automatic differentiation. Differentiating the entire system with respect to just the arguments in X_t alone, I can write the Jacobian of the system with respect to its X_t variables at the non-stochastic steady state as

$$\Gamma_{X,X} \equiv \nabla_X \widehat{\Omega}(\mathbf{0})$$

While the derivatives of the system with respect to the expectation errors and the perturbations are

$$\Gamma_{X,\delta} \equiv \nabla_{d\delta}\Omega(\mathbf{0})$$

$$\Gamma_{X,W} \equiv \nabla_{dW_\zeta}\Omega(\mathbf{0})$$

A first-order Taylor expansion of the system around the steady state without any shocks (and where $d\widehat{X}_t = 0$) is then

$$\Gamma_0 d\widehat{X}_t = \Gamma_{X,X}\widehat{X}_t dt + \Gamma_{X,\delta} d\delta_{X,t} + \Gamma_{X,W} d\epsilon_{\zeta,t} + \mathcal{O}(\|\widehat{X}_t, d\delta_{X,t}, d\epsilon_{\zeta,t}\|^2)$$

I then solve

$$\Gamma_0 d\widehat{X}_t = \Gamma_{X,X}\widehat{X}_t dt + \Gamma_{X,\delta} d\delta_{X,t} + \Gamma_{X,W} d\epsilon_{\zeta,t} \tag{E.20}$$

using the generalized eigenvalue methodology described in Sims (2002). If the system has more stable generalized eigenvalues than it has control variables, the dimensionality of the linear subspace being used to approximate the system's stable manifold is too large to ensure that the dynamics are unique, such that multiple equilibria are possible (sunspots). If the system has fewer stable eigenvalues than state variables, then the equilibrium cannot exist. I verify that the number of stable eigenvalues in my system matches the number of state variables, such that the solution exists and is unique.

While straightforward, this approach is too computationally costly to be feasible with the number of gridpoints that I employ to solve my full model. As such, I use the dimension reduction strategy of Bayer and Luetticke (2020) before calculating the Jacobian of (E.19).

Appendix E.2. With Dimension Reduction

I write the 2-dimensional discrete cosine transform (DCT) of a 2-dimensional array A as $\theta^A = \mathrm{DCT}(A)$, where its inverse $\mathrm{DCT}^{-1}(\theta^A) = A$. I can write the transformation of the value function in the non-stochastic steady state as

$$\{\theta^V_{(i,j),nss}\}_{(i,j)\in I\times J} = \mathrm{DCT}(\{V_{(i,j),nss}\}_{(i,j)\in I\times J})$$

I then compute the "energy" (to use the terminology of Bayer and Luetticke (2020)) of the $\theta^V_{i,j,nss}$ coefficients as

$$E_{ij} = \frac{[\theta^V_{(i,j),nss}]^2}{\sum_{(i,j)\in I\times J}[\theta^V_{(i,j),nss}]^2}$$

Sorting the coefficients by their energy from greatest to least, I then identify those coefficients that contain a cumulative $1-\kappa$ share of the coefficients' energy, where κ is a small number. I label the set of these coefficients (which are effectively the ones with the largest absolute value) as Θ_E; these coefficients explain most of the variation of the value function in the steady state.

As in Bayer and Luetticke (2020), I then move toward constructing a perturbation solution of the equilibrium system, but perturbing only high-energy coefficients in Θ_E. Otherwise, I keep the lower-energy coefficients constant, at their steady state values:

$$\tilde{\theta}^V_{i,j,t} = \theta^V_{(i,j),t} + \mathbf{1}_{\{(i,j)\in\Theta_E\}}\widehat{\theta}^V_{(i,j),t}$$

where $\widehat{\theta}^V_{i,j,t}$ is the coefficient's deviation at time t from its NSS value.

The DCT is a linear operator. As such, I can write the differentials of the coefficients as

$$\{d\theta^V_{(i,j),t}\}_{(i,j)\in I\times J} = d\left[\mathrm{DCT}(\{V_{(i,j),t}\}_{(i,j)\in I\times J})\right] = \{d\theta^V_{(i,j),nss}\}_{(i,j)\in I\times J} = \left[\mathrm{DCT}(\{dV_{(i,j),nss}\}_{(i,j)\in I\times J})\right]$$

and similarly I write

$$d\tilde{\theta}^V_{(i,j),t} = \mathbf{1}_{\{(i,j)\in\Theta_E\}}d\theta^V_{(i,j),t}$$

By perturbing only the $|\Theta_E|$ largest-magnitude coefficients instead of the full $N_a \times N_z$ elements of the discretized value function, I can greatly reduce the dimensionality of the problem. Of course, this only reduces the number of control variables. To reduce the number of state variables in the distribution, I also employ the fixed copula transformation of Bayer and Luetticke (2020).

I write the discretized joint cumulative distribution function $F_{\mu(a_i,z_j)}$, and the marginal CDFs as $F_{\mu(a_i)}$ and $F_{\mu(z_j)}$. The copula is then the joint distribution interpolated onto the marginal ones:

$$\text{Cop} = \text{Interp}(\{F_{\mu(a_i,z_j),nss}\}_{ij}, \{F_{\mu(a_i),nss}\}_i, F_{\mu(z_j),nss}\}_j)$$

where the nss subscript denotes the steady state values. It then follows that $\text{Cop} : [0,1] \times [0,1] \to [0,1]$ maps cumulative marginal distributions to a joint distribution, as predicted by the rank correlations of the steady state. Outside of the steady state, I then approximate the joint cumulative distribution $F_{\mu(a_i,z_j),t}$ at time t as

$$F_{\mu(a_i,z_j),t} \approx \text{Cop}(F_{\mu(a_i),t}, F_{\mu(z_j),t}),$$

from which the marginal joint *density* function μ_{ij} may be derived. Using this object, I can then iterate the Kolmogorov Forward Equation to obtain $d\mu_{ij}$, which can be integrated (or summed, since the functions are discretized) to obtain the evolution of the differentials

$$\{(dF_{\mu(a_i),t}, dF_{\mu(z_j),t})\}_{ij}.$$

As Bayer and Luetticke (2020) note, this approximation allows me to track only the N_a and N_z dimensional marginal CDFs instead of their joint one to describe the economy, so long as the rank correlations outside of the steady state are similar to those represented in the steady state (which Bayer and Luetticke (2020) show is generally the case in Bewley-Aiyagari models).

I then define the dimension-reduced set of controls as

$$\tilde{X}_{C,t} = (\{\tilde{\theta}^V_{i,j,t}\}_{(i,j)\in\Theta_E},\ \pi_t,\ \pi^w_t,\ q_t)'$$

and the dimension-reduced set of states as

$$\tilde{X}_{1,t} = (\{F_{\mu(a_i),t}\}_i, \{F_{\mu(z_j),t}\}_j,\ B_t,\ w_t, \zeta_t)',$$

Once again stacking the reduced controls, states, and static variables, I write

$$\tilde{X}_t = (\tilde{X}_{C,t}, \tilde{X}_{1,t}, X_{L,t})'$$

and the system (E.18) is approximated by a smaller one:

$$\tilde{\Gamma}_0 d\tilde{X}_t = \tilde{\Omega}(\tilde{X}_t, d\delta_{X,t}, d\epsilon_{\zeta,t})$$

where $\tilde{\Omega}$ calculates the value function and joint distribution given the DCT coefficients and the marginal distribution, feeds them back into the original Ω function, and then from there recovers the resulting truncated DCT coefficients and marginal CDFs' time differentials. Just like before, this system can also be written in terms of just the differences (or log differences) of the variables from their non-stochastic steady state values. The rest of the linearization steps and solution methods then proceed exactly in the same manner as they do in the version without dimension reduction, as reviewed in the prior subsection of this appendix.

I solve the model over a uniform grid of $N_a = 100$ points spaced nonlinearly from 0 to 60, with more grid points at the bottom of the asset distribution. I use $N_z = 50$ grid points from 0.01 to 5.5.